Aalto University
School of Science

Lecture 2: Quantum geometry and superconductivity: recent developments

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Aalto University

Topological Matter School 2024, Donostia-San Sebastián

22.8.2024

SuperC Room Temperature Superconductivity 2033

Institute Q

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HORIZON2020 Marie Skłodowska-Curie actions

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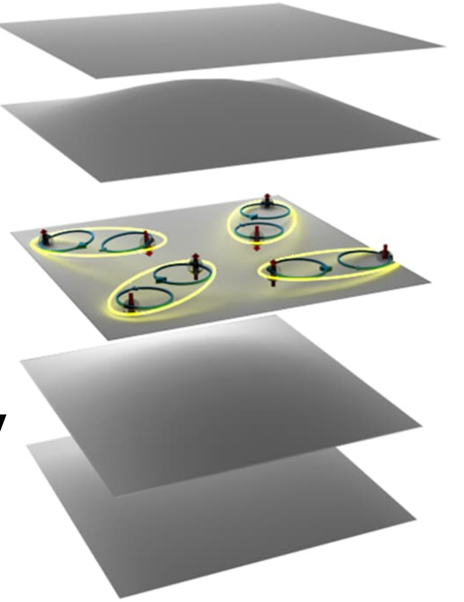

Lecture 1

- Basics of quantum geometry
- Quantum geometry and superconductivity

Lecture 2

- Flat band superconductivity and quantum geometry in twisted bilayer graphene (TBG)
- Non-Fermi liquid normal states in flat bands
- Non-equilibrium transport in flat band superconductors
- DC conductivity in a flat band
- The many-body quantum metric and the Drude weight

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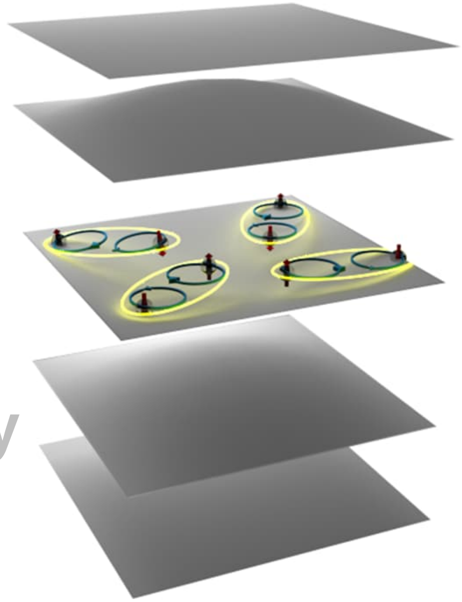

Lecture 1

- Basics of quantum geometry
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Lecture 2

- **Flat band superconductivity and quantum geometry in twisted bilayer graphene (TBG)**
- Non-Fermi liquid normal states in flat bands
- Non-equilibrium transport in flat band superconductors
- DC conductivity in a flat band
- The many-body quantum metric and the Drude weight

Twisted Bilayer Graphene (TBG) superconductivity since 2018

Reviews: Balents, Dean, Efetov, Young, Nat Phys 2020
Andrei, Efetov, Jarillo-Herrero, MacDonald, Mak, Senthil, Tutuc, Yazdani, Young, Nat Rev Mater 2021

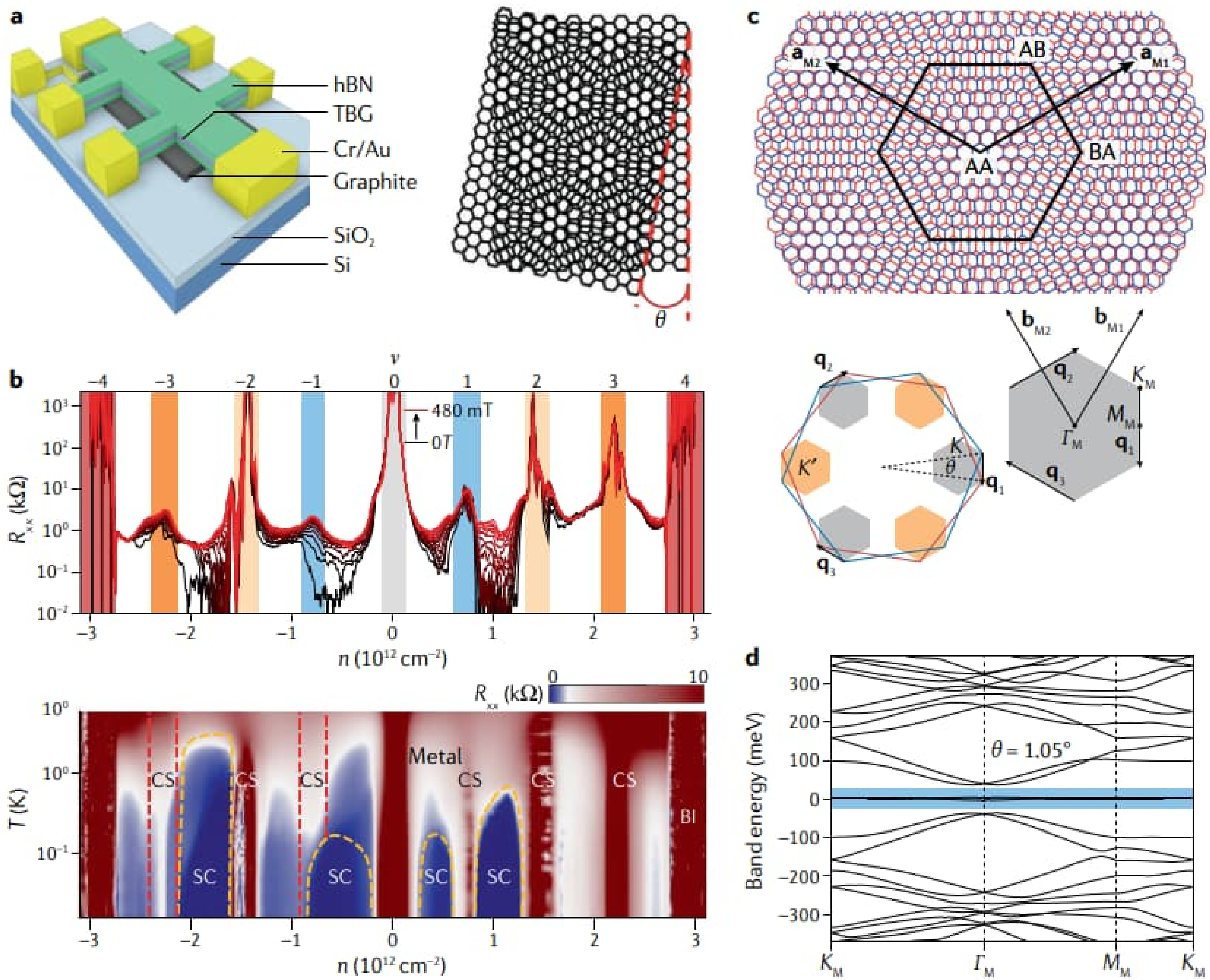
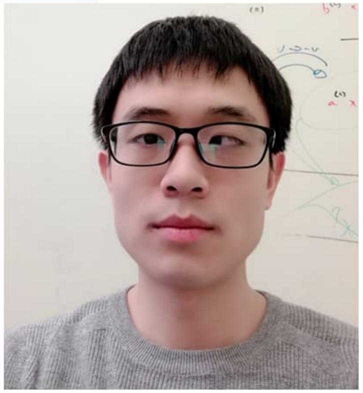


Figure credits see Fig.1 in PT, Peotta, Bernevig, Nat Rev Phys 2022

Geometric contribution in TBG superconductivity

Aleksi Julku

Teemu Peltonen



Long Liang

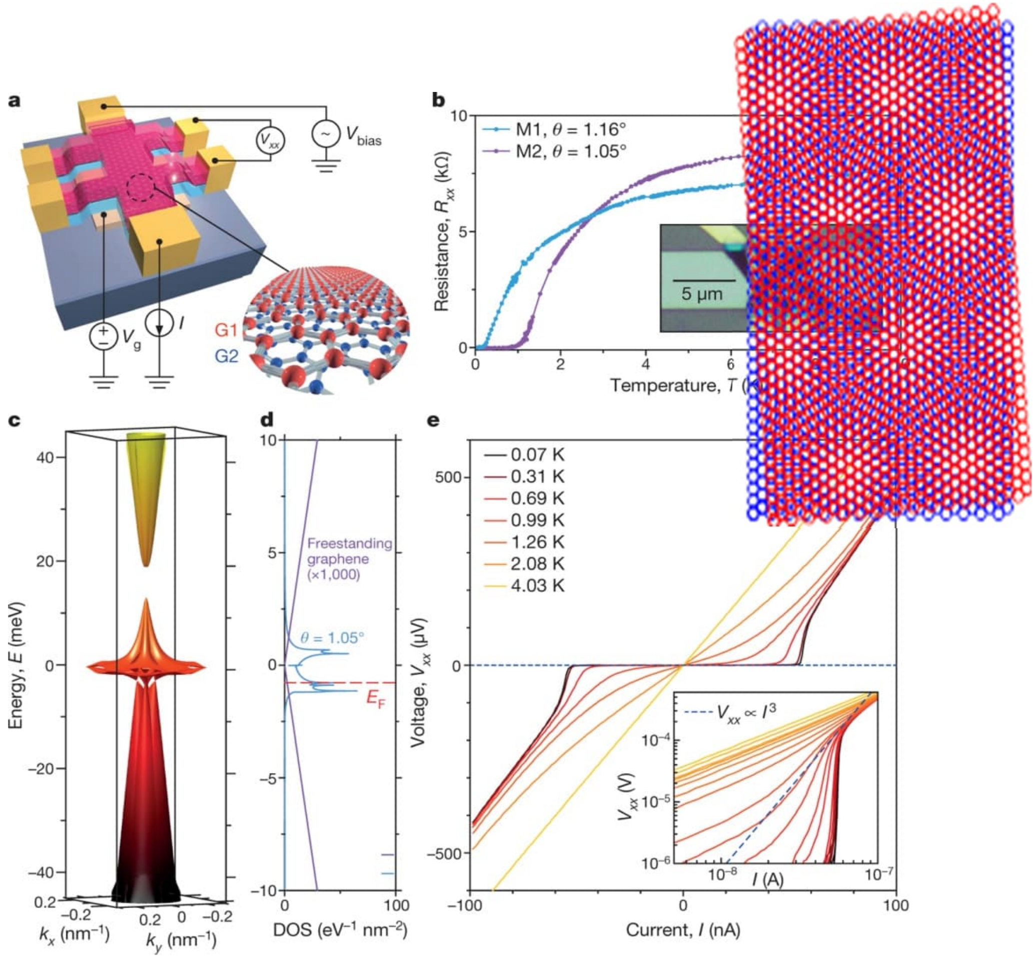
Tero Heikkilä

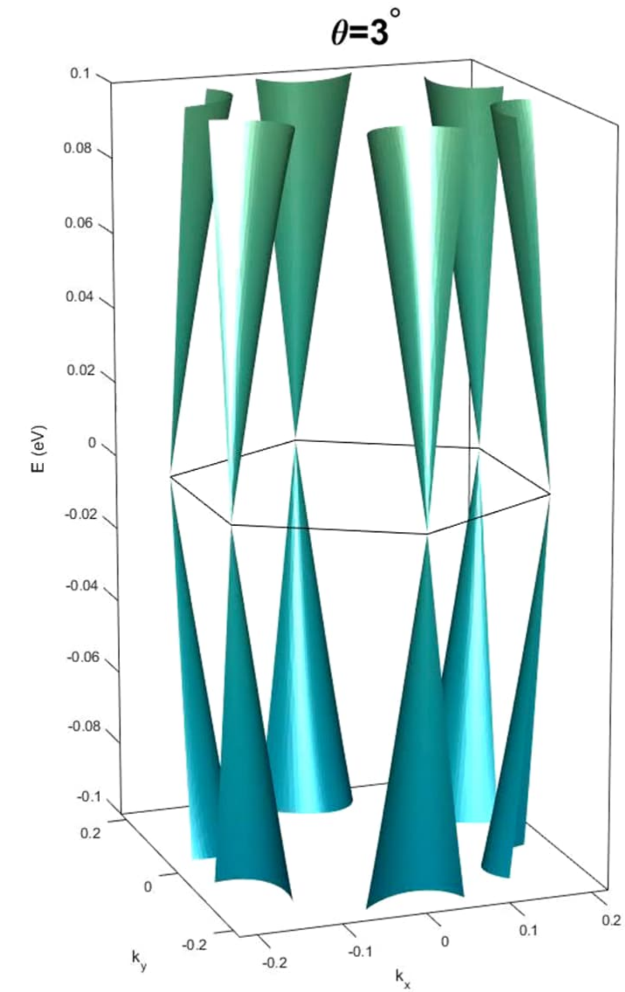
Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors' Suggestion

MA-TBG: Magic **A**ngle-**T**wisted **B**ilayer **G**raphene

Twisting graphene layers produces flat bands

(unconventional) superconductivity





Y Cao *et al.* *Nature* **556**, 43–50 (2018)

Also
Nature 556, 80 (2018)
Science 363, 1059 (2019)
Nature 574, 653-657 (2019))

VIEWPOINT

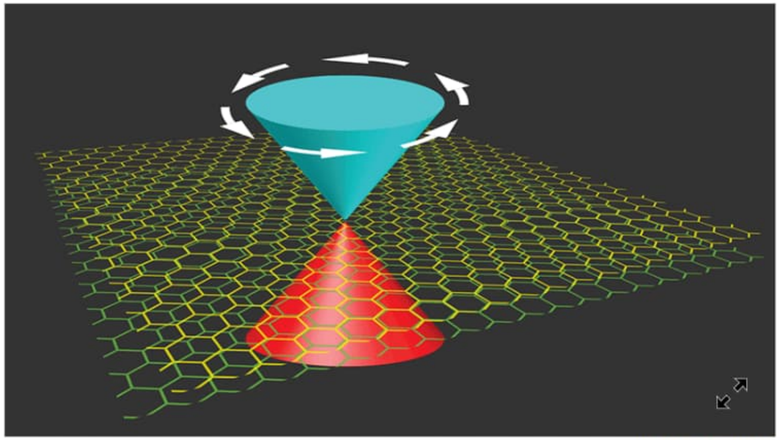
PDF Version

Geometry Rescues Superconductivity in Twisted Graphene

Laura Classen
School of Physics and Astronomy, University of Minnesota, Minneapolis, MN, USA

February 24, 2020 • *Physics* 13, 23

Three papers connect the superconducting transition temperature of a graphene-based material to the geometry of its electronic wave functions.



APS/Alan Stonebraker

Figure 1: Electrons moving through the sheets of twisted bilayer graphene (TBG) have special points in their band structure where two cone-shaped bands meet. The inherent "curvature" of the states in these bands turns out to contribute to the magnitude of TBG'... Show more

On its own, a sheet of graphene is a semimetal—its electrons interact only weakly with each other. But as experimentalists discovered in 2018 [1, 2], the situation changes when two sheets of graphene are stacked together, with a slight ($\sim 1°$) rotation between them (Fig. 1). At this so-called magic twist angle [3] and at low temperatures [1], the electrons become correlated, forming insulating or superconducting phases depending on the carrier density [2–7]. These phases appear to come from a twist-induced flattening of the electronic energy bands, which

Geometric and Conventional Contribution to the Superfluid Weight in Twisted Bilayer Graphene
Xiang Hu, Timo Hyart, Dmitry I. Pikulin, and Enrico Rossi
Phys. Rev. Lett. 123, 237002 (2019)
Published December 5, 2019
Read PDF

Superfluid weight and Berezinskii-Kosterlitz-Thouless transition temperature of twisted bilayer graphene
A. Julku, T. J. Peltonen, L. Liang, T. T. Heikkilä, and P. Törmä
Phys. Rev. B 101, 060505 (2020)
Published February 24, 2020
Read PDF

Topology-Bounded Superfluid Weight in Twisted Bilayer Graphene
Fang Xie, Zhida Song, Biao Lian, and B. Andrei Bernevig
Phys. Rev. Lett. 124, 167002 (2020)
Published April 24, 2020
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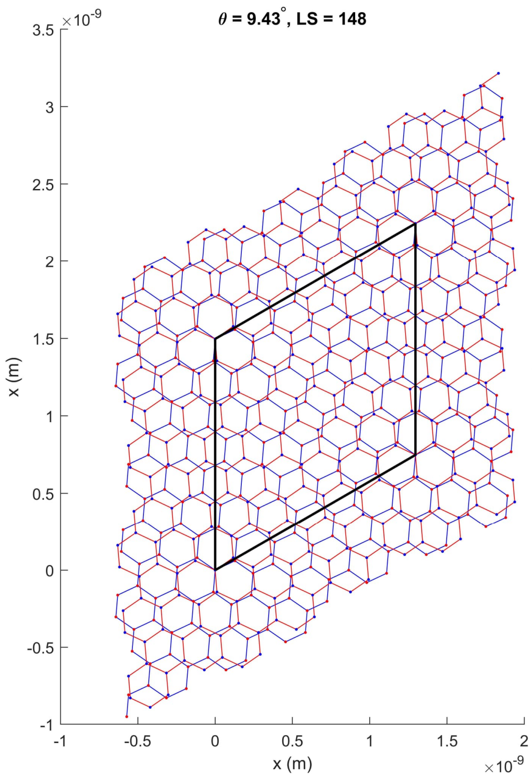
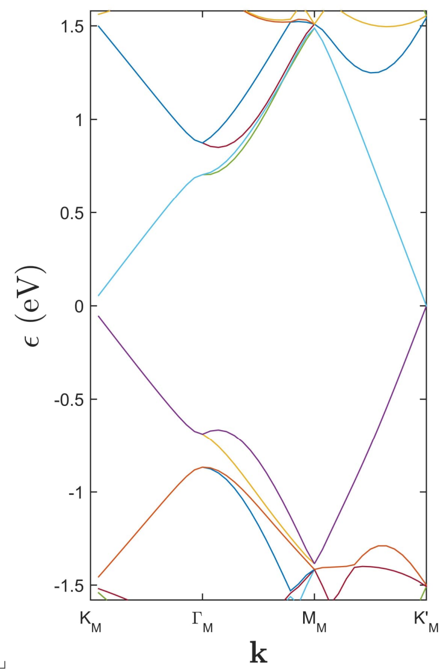
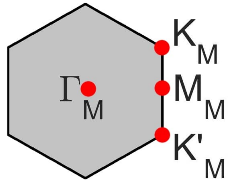
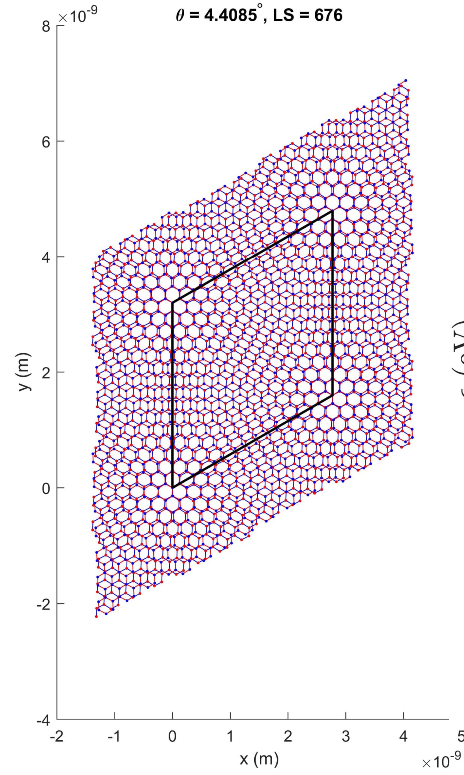
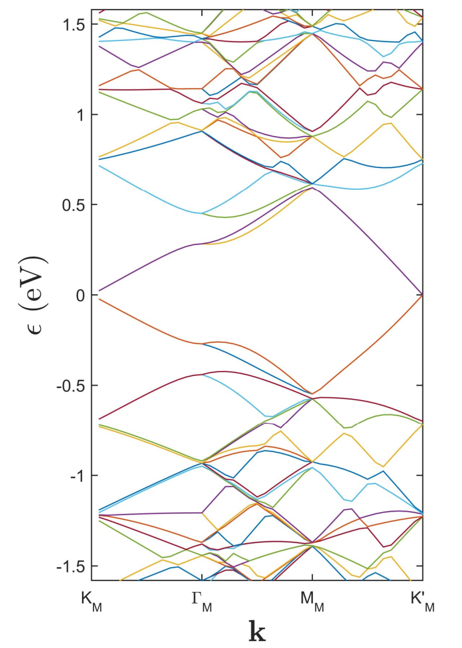
Non-interacting bands



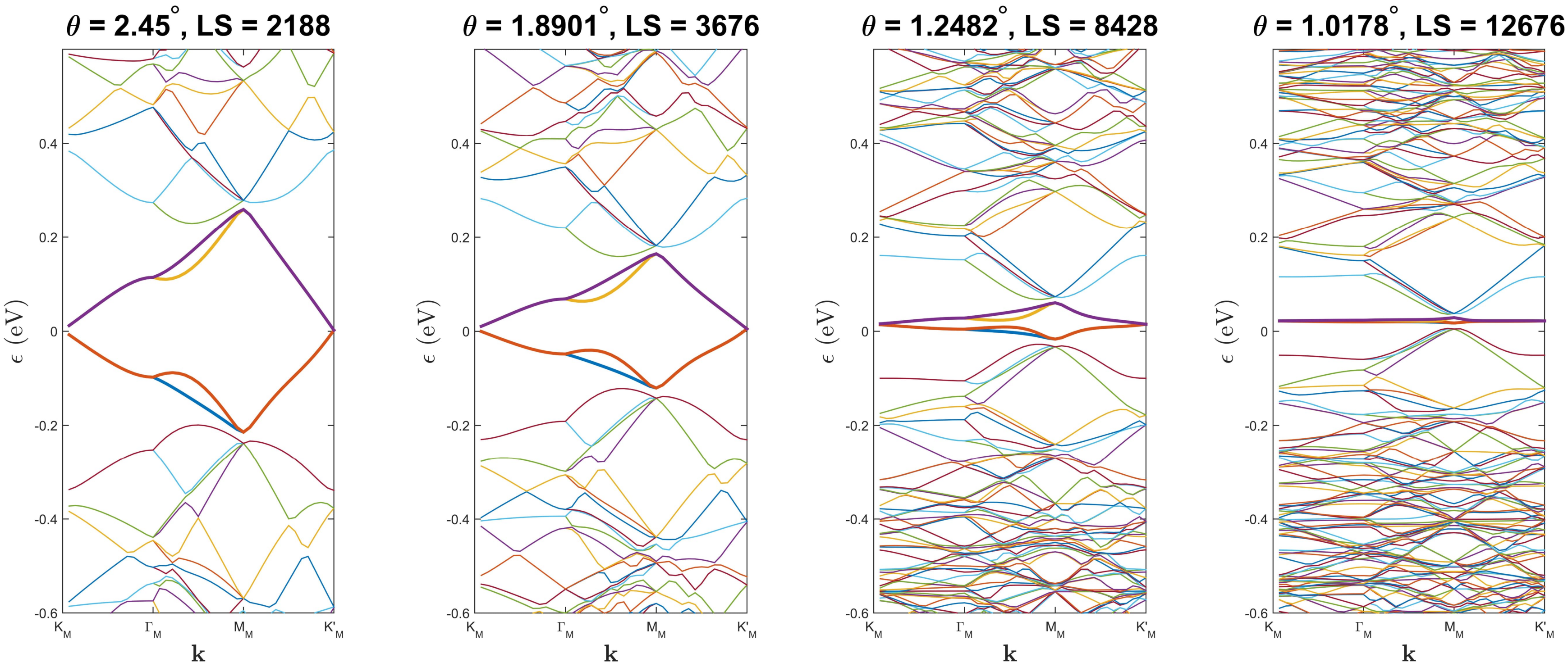








Non-interacting bands



At magic angle θ ~1 deg, the number of lattice sites per unit cell (LS) around 13 000: numerically still a problem even at the mean-field level

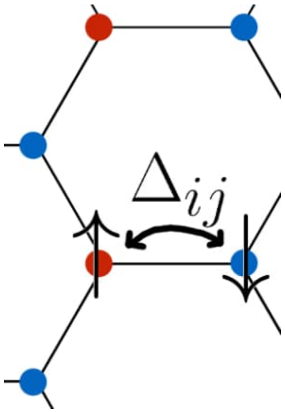
We reduce LS to around 700 by applying a rescaling trick which modifies the twist angle but keeps the Moire periodicity and the Dirac velocity invariant

Fermi-Hubbard lattice model with TBG geometry (600 bands)

$$H = \sum_{ij\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + H_{\text{int}}$$

Two pairing schemes

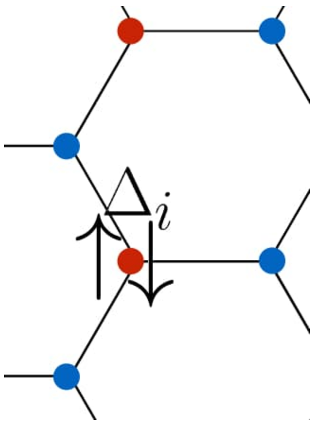
Non-local (RVB) interaction



$$H_{\text{int}} = \frac{J}{2} \sum_{\langle ij \rangle} h^\dagger_{ij} h_{ij}$$

$$h_{ij} = c_{i\downarrow} c_{j\uparrow} - c_{i\uparrow} c_{j\downarrow}$$

Local (s-wave) interaction



$$H_{\text{int}} = J \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow}$$

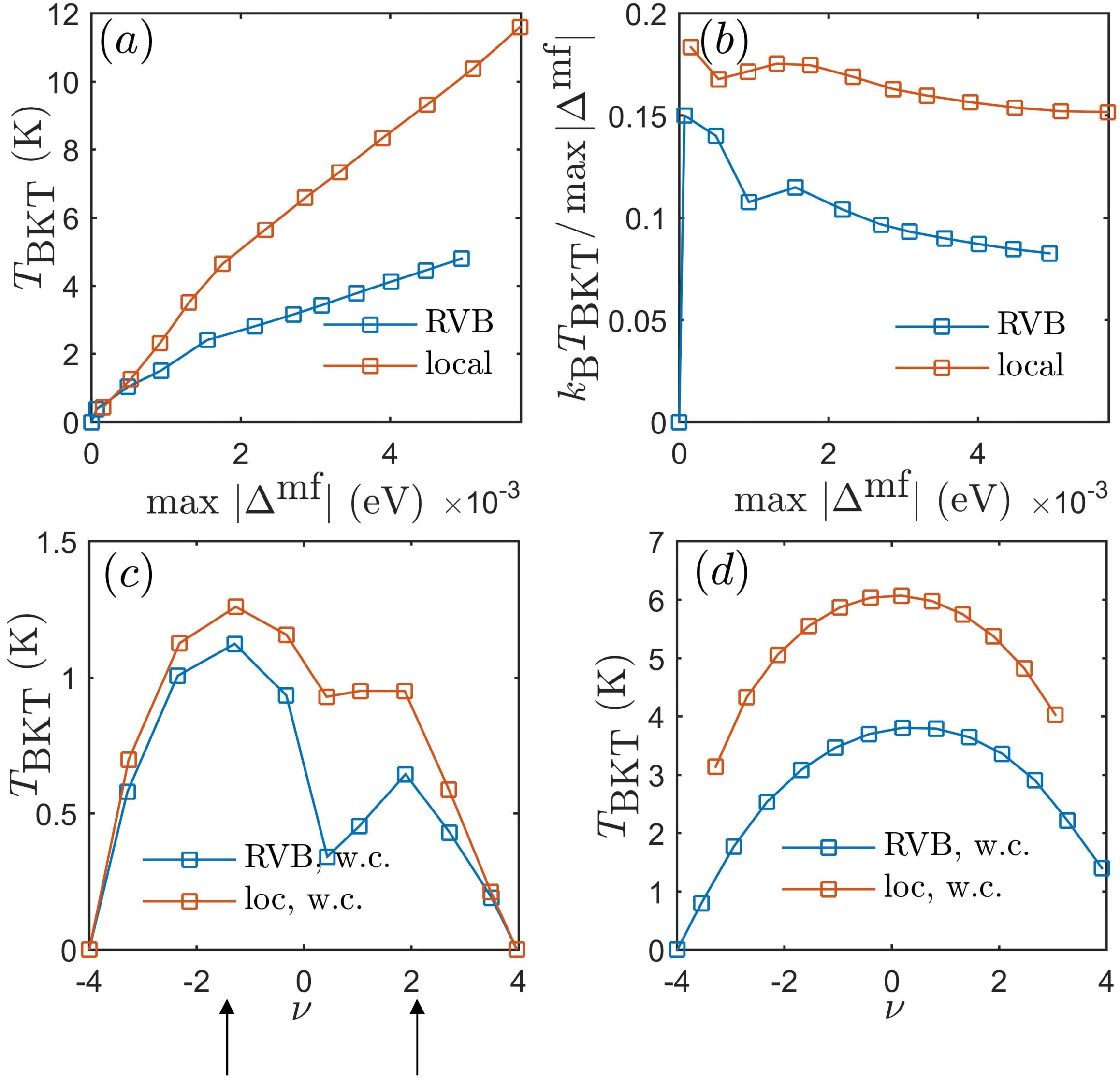
J< 0 is attractive interaction strength

BKT temperature

$$T_{\mathrm{BKT}} = \frac{\pi}{8}\sqrt{\det D^s(T_{\mathrm{BKT}})}$$

For flat band regime local interaction has considerably larger T_{BKT}

Here RVB (resonance valence bond) is the non-local pairing scheme



Small interaction

Large interaction

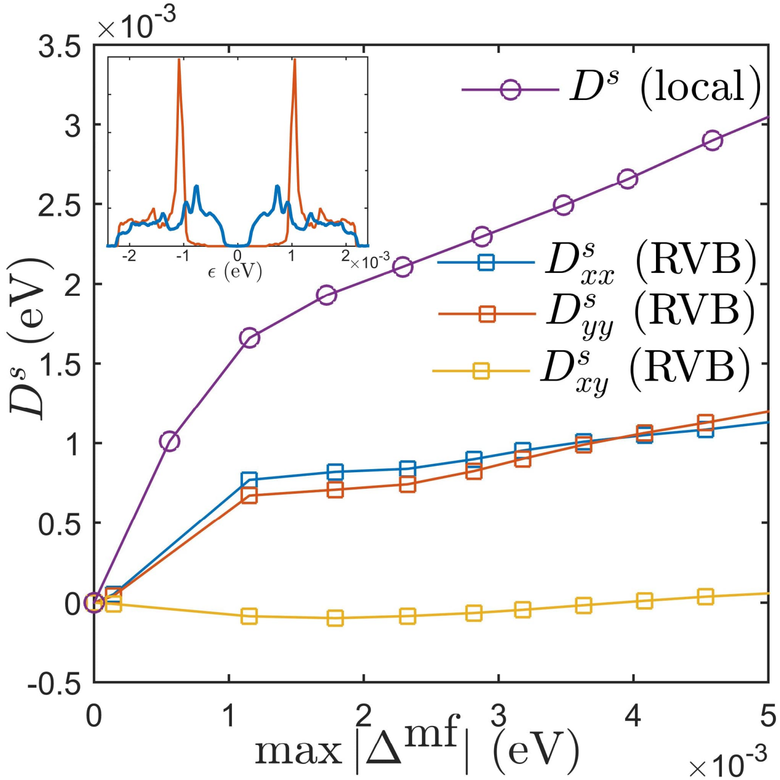
Nematic order parameter for non-local pairing

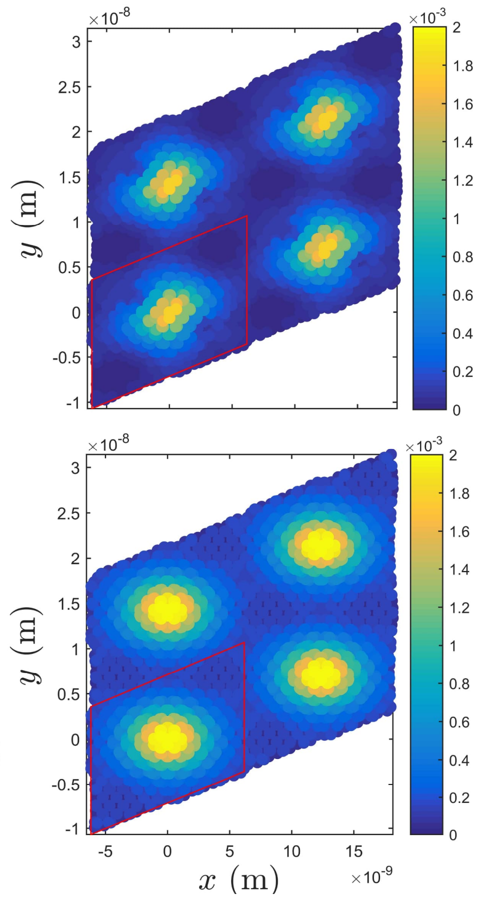
$$D^s = \begin{bmatrix} D^s_{xx} & D^s_{xy} \approx 0 \\ D^s_{yx} \approx 0 & D^s_{yy}, \end{bmatrix}$$

Local pairing preserves the lattice symmetries and yields isotropic D^s

Non-local pairing breaks the rotational symmetry and yields non-isotropic response

Local pairing has s wave symmetry, non-local yields mixed s+p+d symmetry (d dominant)

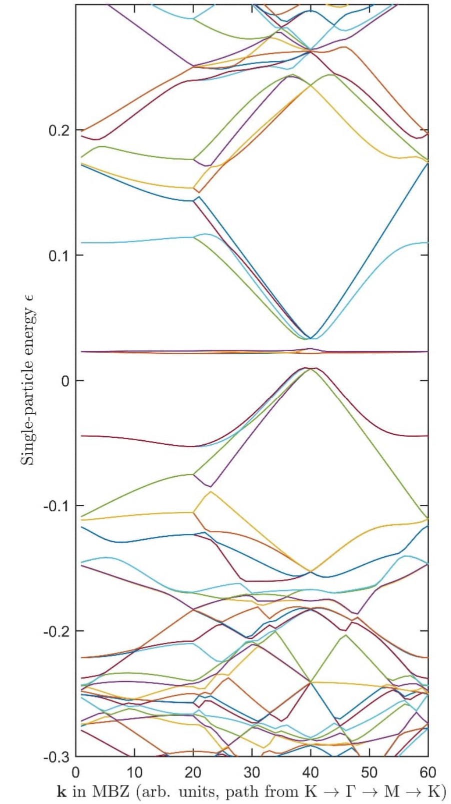


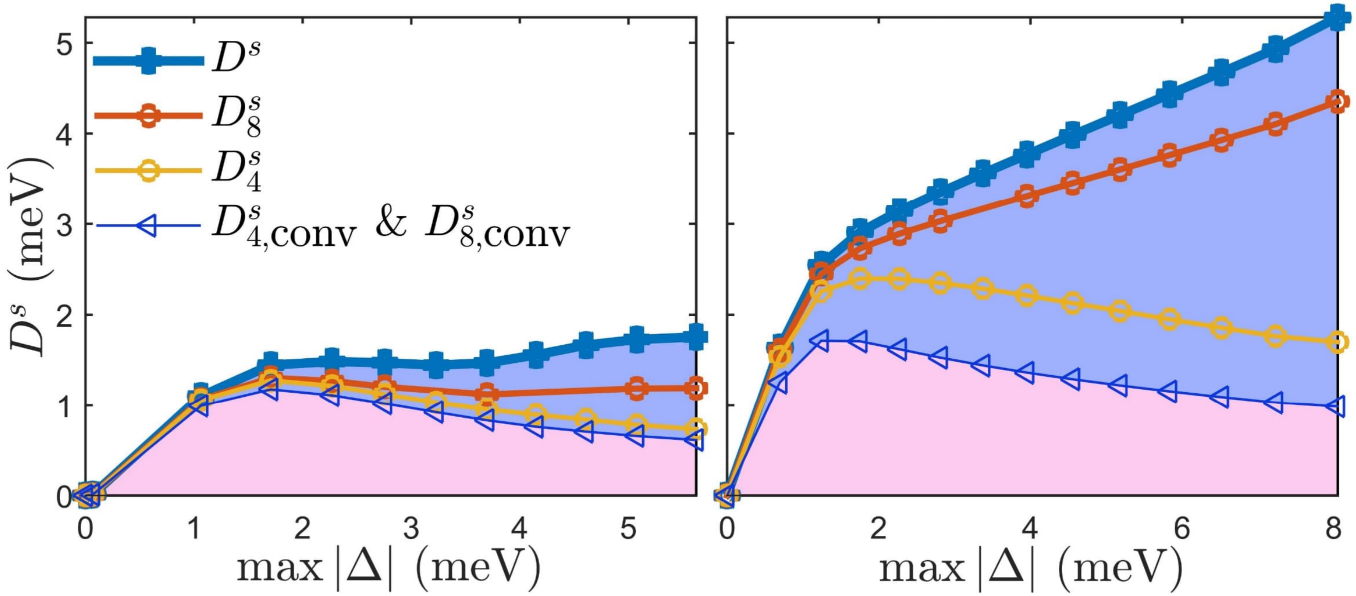


Geometric contribution in TBG

$$D^s = D^s_{\mathrm{conv}} + D^s_{\mathrm{geom}}$$

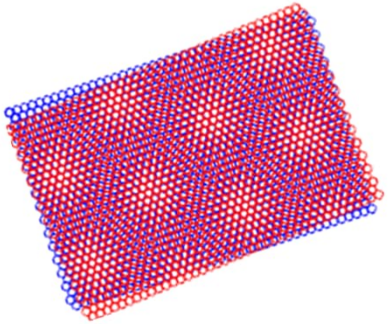
$$T_{\mathrm{BKT}} = \frac{\pi}{8}\sqrt{\det D^s(T_{\mathrm{BKT}})}$$





Non-local (RVB) interaction

Local (s-wave) interaction



Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors' Suggestion

Confirmed by (only s-wave): Hu, Hyart, Pikulin, Rossi, PRL (2019)

Euler class bound of TBG superconductivity: Xie, Song, Lian, Bernevig, PRL (2020)

TBG theory has advanced since 2020 (e.g. Kang, Vafek, PRB 2023; Vafek, Kang, PRB 2023); quantitative predictions to be revisited

First experiments exploring quantum geometric superconductivity in TBG

Tian, Gao, Che, Xu, Cheung, Watanabe, Taniguchi, Randeria, Zhang, Lau, Bockrath, Nature 2023

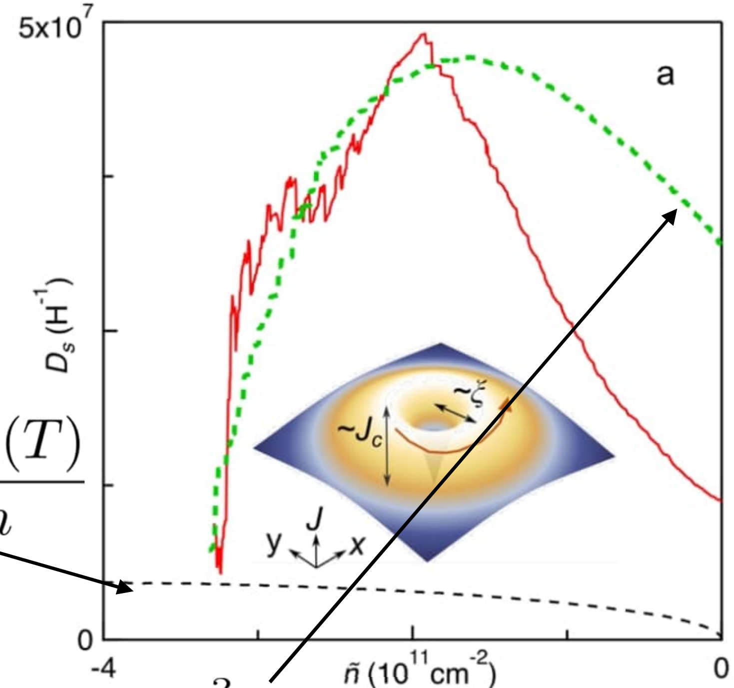
$$\xi = \sqrt{\frac{\Phi_0}{2\pi B_{c2}}} \quad \Phi_0 = \frac{h}{2e} \quad J_{\rm cs} = n_s e \frac{\Delta}{\hbar k_{\rm F}}$$

Critical field and current measured as well as Fermi velocity

Superfluid weight from

$$D_s(0) = \frac{2\pi J_{\rm cs}\xi}{\Phi_0}$$

$$D_s(T) = \frac{e^2 n_s(T)}{m}$$



Isolated flat band

$$[D_s]_{ij} = \frac{2}{\pi\hbar^2}\frac{\Delta^2}{U N_{\rm orb}}\mathcal{M}^{\rm R}_{ij}$$

$$D_s(0,\tilde{n}) \approx b\frac{e^2}{\hbar^2}\Delta(0,\tilde{n})$$

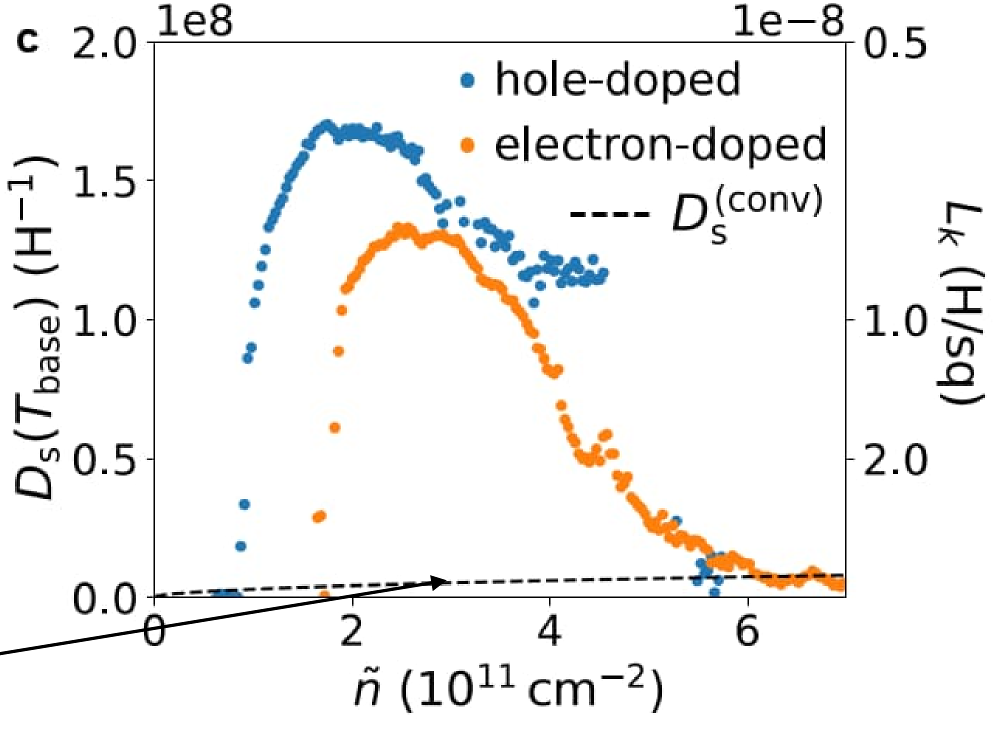
Direct measurement of the kinetic inductance in TBG

Tanaka, Wang, Dinh, Rodan-Legrain, Zaman, Hays, Kannan, Almanakly, Kim, Niedzielski, Serniak, Schwartz, Watanabe, Taniguchi, Grover, Orlando, Gustavsson, Jarillo-Herrero, Oliver, arXiv:2406.13740 (2024)

Superfluid weight is inversely proportional to kinetic inductance

Isolated flat band

$$[D_s]_{ij} = \frac{2}{\pi\hbar^2}\frac{\Delta^2}{UN_{\rm orb}}\mathcal{M}^{\rm R}_{ij}$$



Single band BCS

$$D_s(T) = \frac{e^2 n_s(T)}{m}$$

Flat-band ratio and quantum metric in the superconductivity of modified Lieb lattices
- how general are the isolated flat band results?

How does flat band superfluid weight scale with temperature?

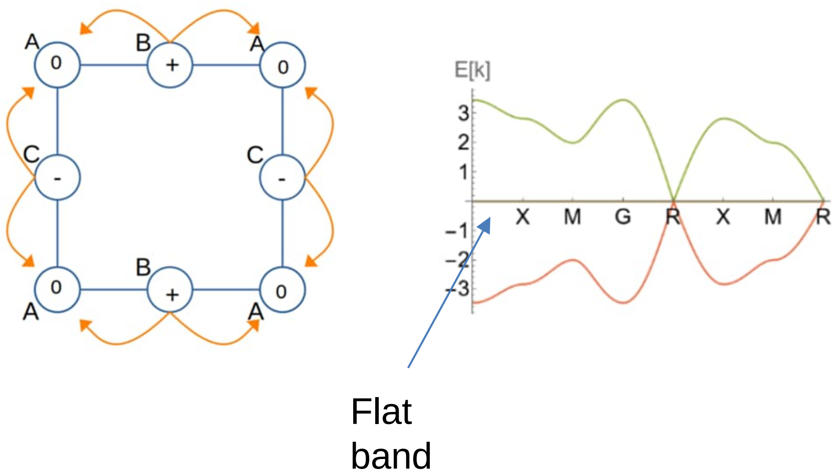
Reko Penttilä

Kukka-Emilia Huhtinen

Penttilä, Huhtinen, PT, arXiv:2404.12993 (2024)

Bipartite flat bands

Lieb lattice



Bipartite lattices have $N_L - N_S$ flat bands
Localized states only on the larger sublattice L

Calugaru… Bernevig, Nature Physics 2022

Isolated flat band results

$$[D_s]_{ij} = \frac{4U\nu(1-\nu)N_f}{(2\pi)^{d-1}N_{\mathrm{orb}}}\mathcal{M}_{ij}^{\mathrm{R,min}}$$

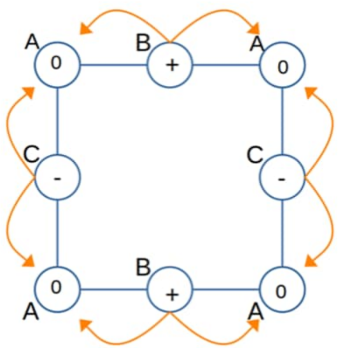
$$\mathcal{M}_{ij}^{\mathrm{R}} = \frac{1}{2\pi}\int_{\mathrm{B.Z.}} \mathrm{d}^d\mathbf{k}\,\underbrace{\mathrm{Re}\,\mathcal{B}_{ij}(\mathbf{k})}_{g_{ij}}$$

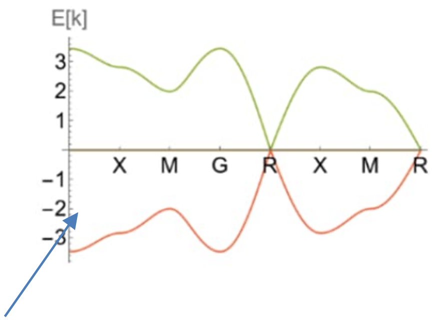
Zero temperature superfluid weight given by the minimal quantum metric and the flat-band ratio

Peotta, PT, Nat Comm 2015
Huhtinen, Herzog-Arbeitman, Chew, Bernevig, PT, PRB 2022
Herzog-Arbeitman, Chew, Huhtinen, PT, Bernevig, arXiv 2022

Bipartite flat bands

Lieb lattice





Flat band

Bipartite lattices have $N_L - N_S$ flat bands

Localized states only on the larger sublattice L

Calugaru… Bernevig, Nature Physics 2022

$$D_{\mu\nu} = \frac{4f(1-f)}{(2\pi)^{D-1}} \frac{N_f}{N_{of}} |U| M^{\min}_{\mu\nu}$$

$$\mathcal{M}^{\mathrm{R}}_{ij} = \frac{1}{2\pi} \int_{\mathrm{B.Z.}} \mathrm{d}^d\mathbf{k} \, \underbrace{\mathrm{Re}\, \mathcal{B}_{ij}(\mathbf{k})}_{g_{ij}}$$

Zero temperature superfluid weight given by the minimal quantum metric and to the flat-band ratio

Peotta, PT, Nat Comm 2015

Huhtinen, Herzog-Arbeitman, Chew, Bernevig, PT, PRB 2022

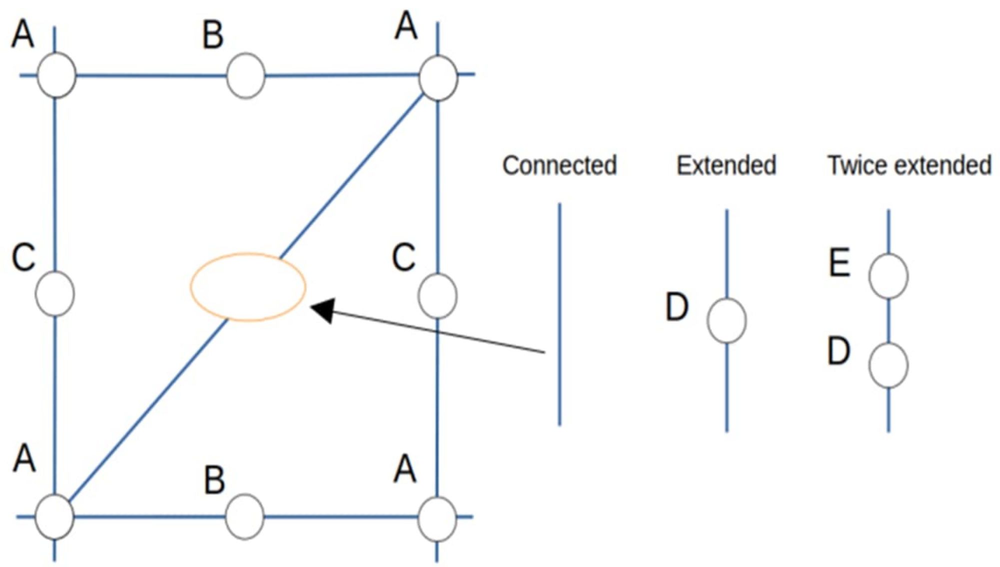
Herzog-Arbeitman, Chew, Huhtinen, PT, Bernevig, arXiv 2022

Modified Lieb lattices

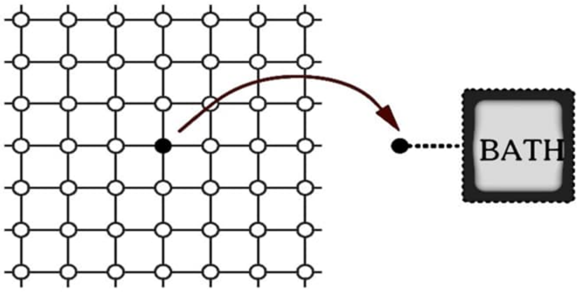
Modifications change the number of flat bands and orbitals

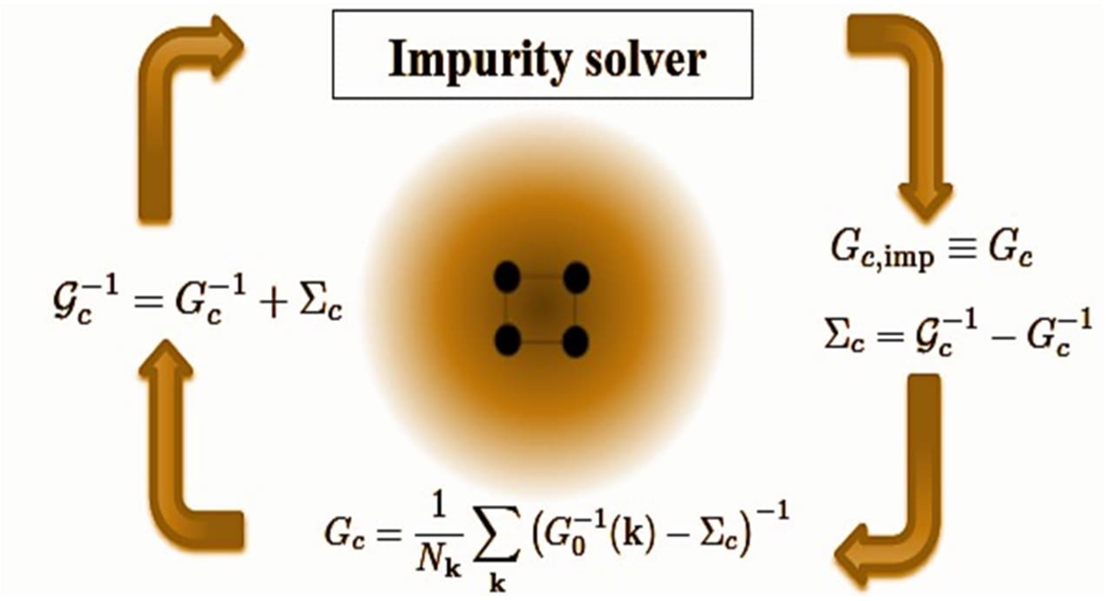
$$[D_s]_{ij} = \frac{4U\nu(1-\nu)N_f}{(2\pi)^{d-1}N_{\mathrm{orb}}}\mathcal{M}_{ij}^{\mathrm{R,min}}$$

Lattice	N_{of}	N_f
Lieb	2	1
Connected	2	1
Extended	3	2
Twice extended	2	1



Dynamical Mean Field Theory (DMFT) to capture quantum effects *beyond mean-field*





Single site DMFT

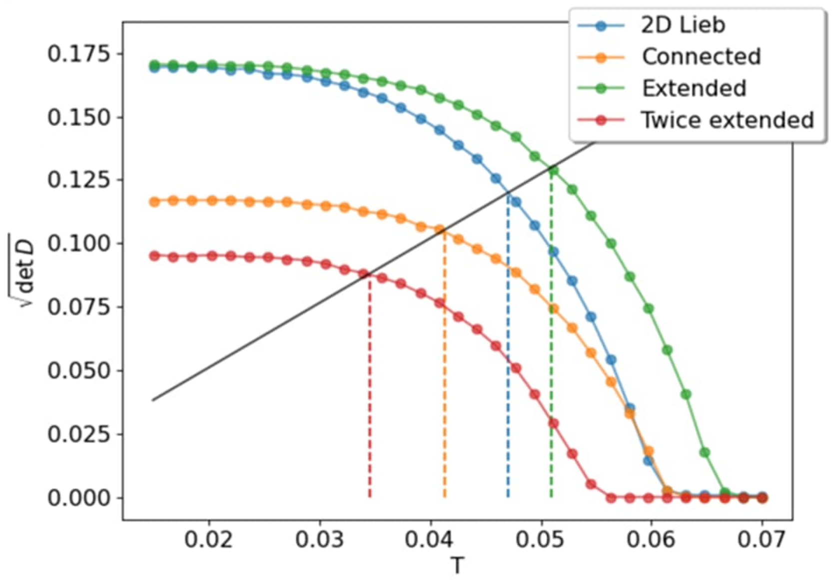
Cellular/cluster DMFT; Non-local correlations

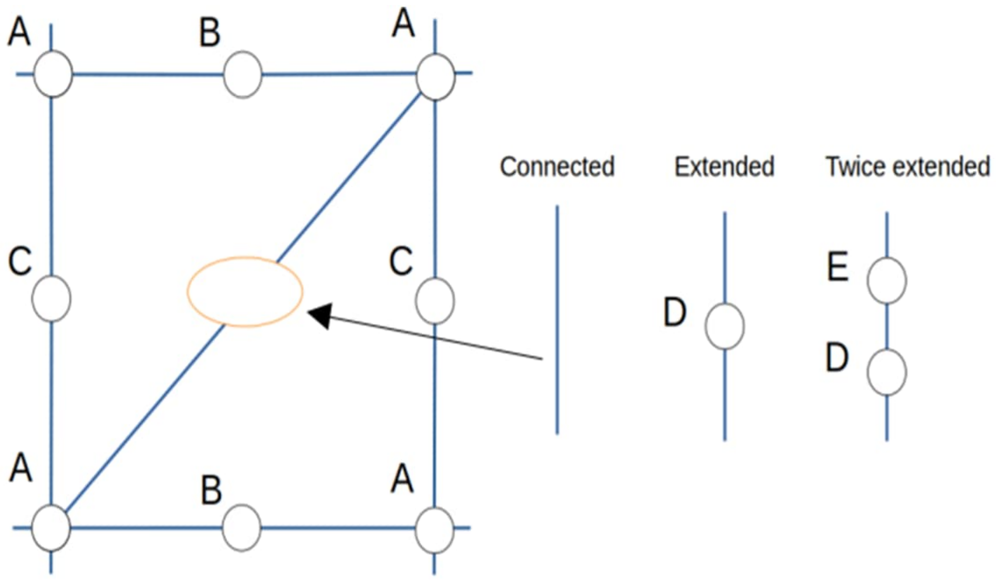
Superfluid weight from DMFT

Superfluid weight from DMFT Green's functions

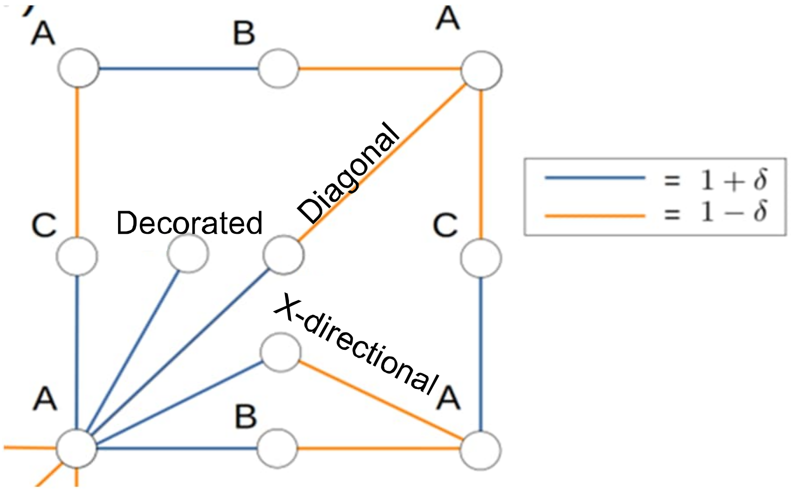
$$\langle j_\mu \rangle = D^s_{\mu\nu} A_\nu \qquad t_{ij} \to e^{iA\cdot(r_i - r_j)} t_{ij}$$

Extended lattice has both the largest critical temperature and flat-band ratio



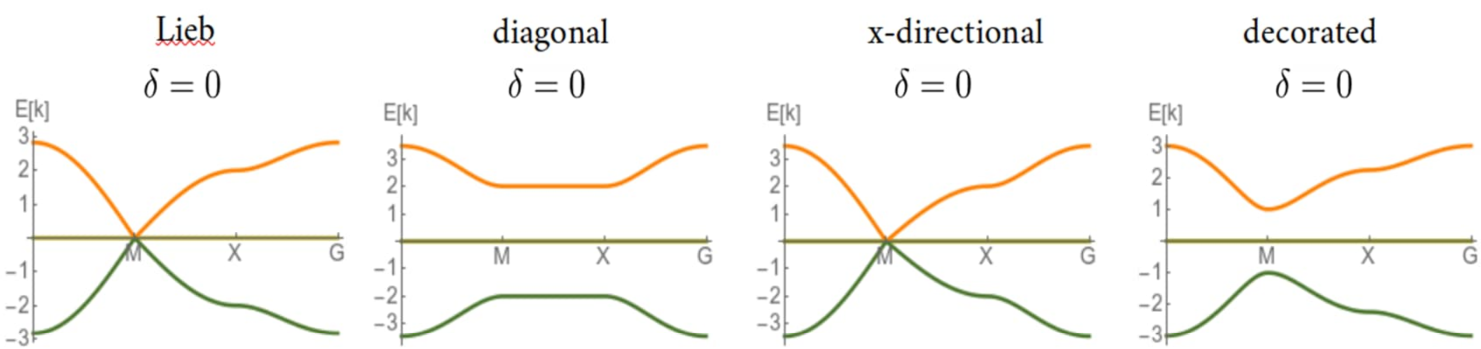


Different extended Lieb lattices

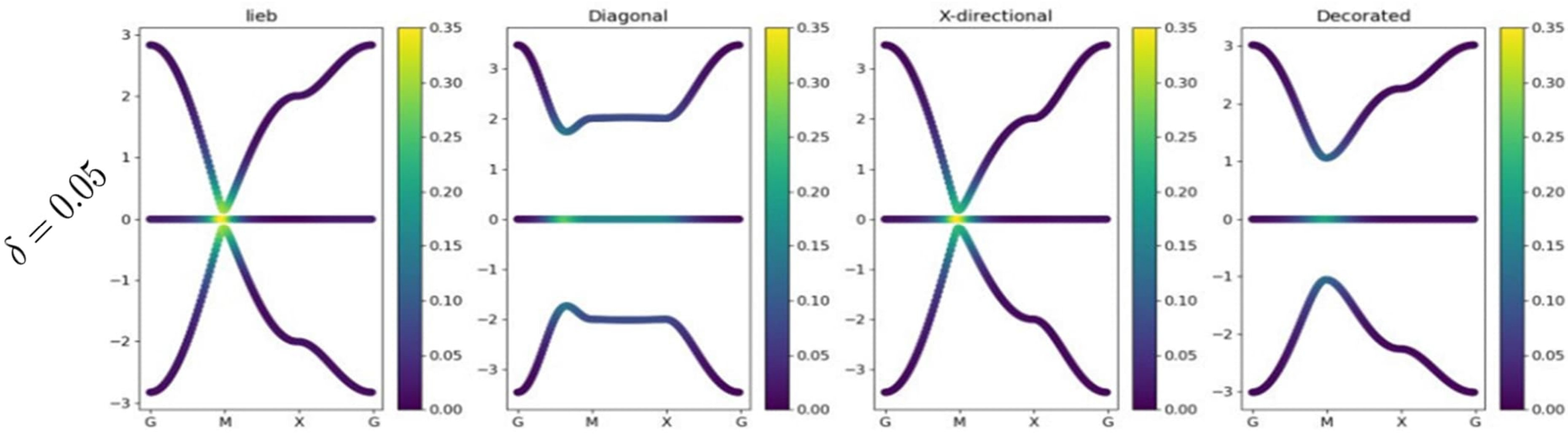


$$[D_s]_{ij} = \frac{4U\nu(1-\nu)N_f}{(2\pi)^{d-1}N_{\mathrm{orb}}}\mathcal{M}_{ij}^{\mathrm{R,min}}$$

1/2 2/3 2/3 2/3

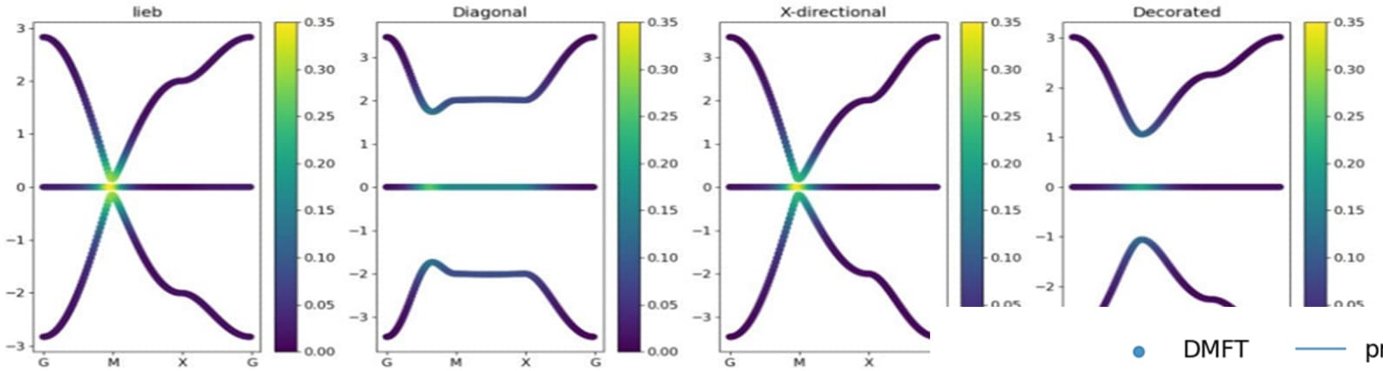


Quantum metric of the bands

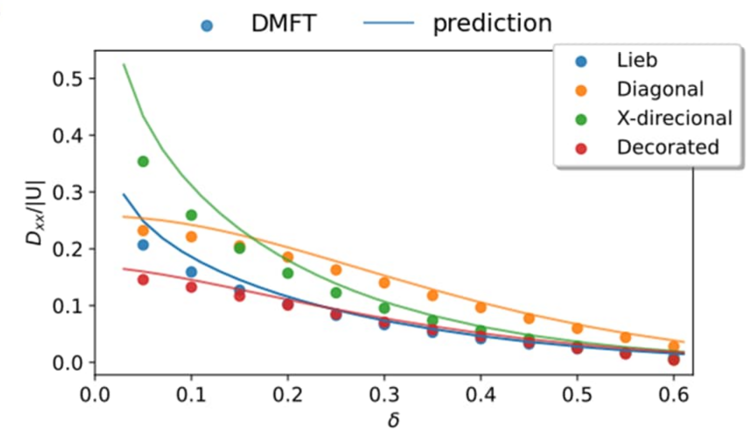


Quantum metric determines the superfluid weight also in DMFT results

$\delta = 0.05$

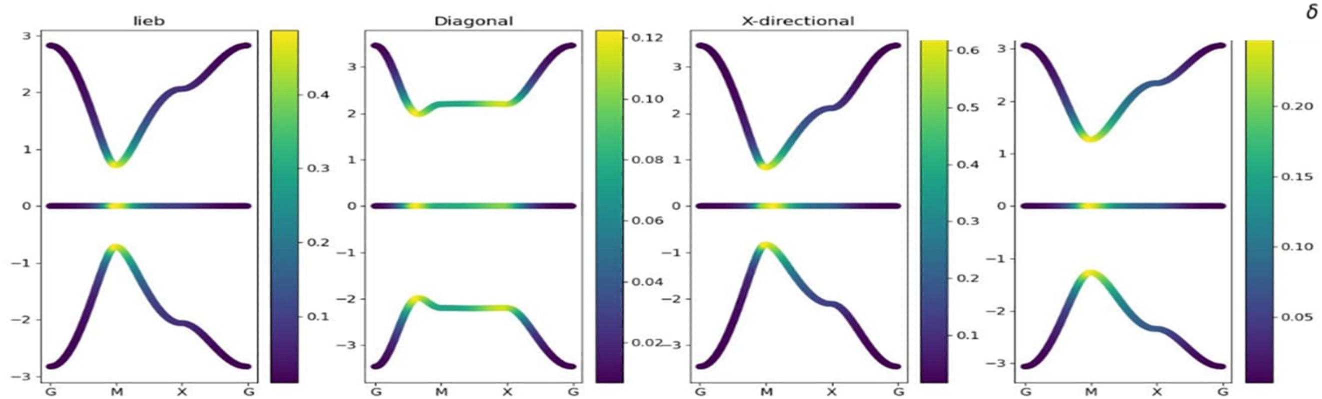


$$[D_s]_{ij} = \frac{4U\nu(1-\nu)N_f}{(2\pi)^{d-1}N_{\mathrm{orb}}}\mathcal{M}_{ij}^{\mathrm{R,min}}$$

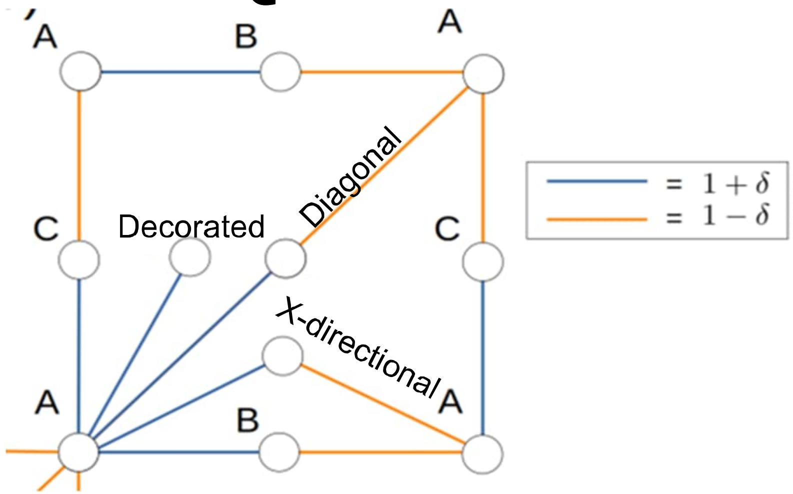


$|U| = 0.3$

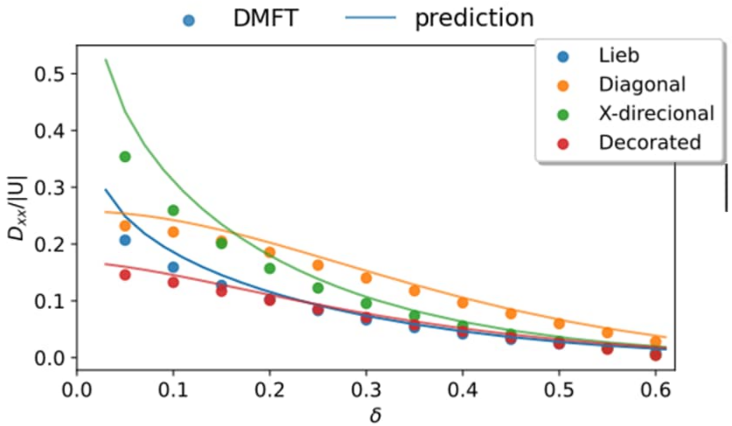
$\delta = 0.3$



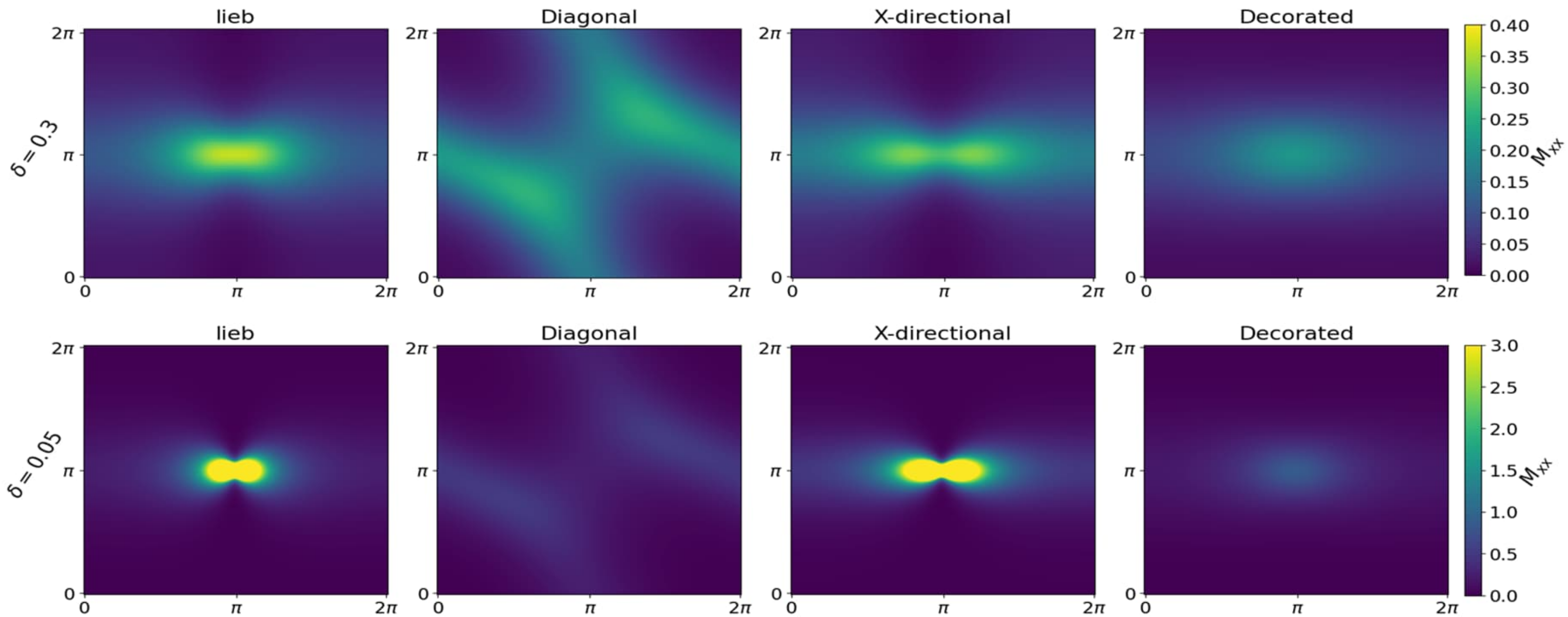
Quantum metric over the whole BZ



$$[D_s]_{ij} = \frac{4U\nu(1-\nu)N_f}{(2\pi)^{d-1}N_{\rm orb}}\mathcal{M}^{\rm R,min}_{ij}$$



$|U| = 0.3$

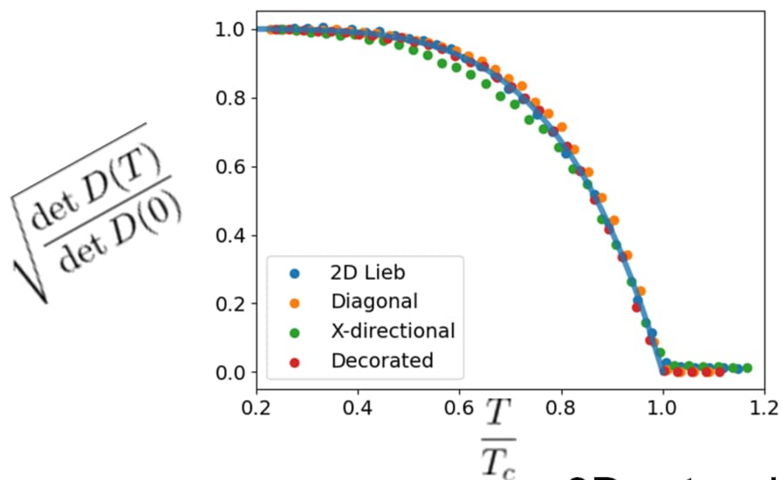


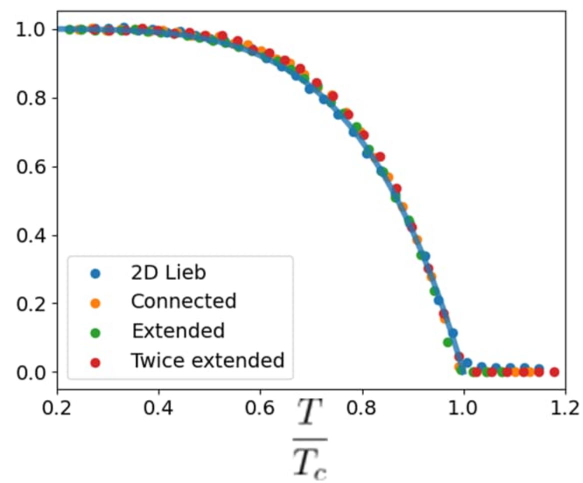
Qualitative behaviour of the superfluid weight

All lattices have the same behavior of the superfluid weight as a function of temperature

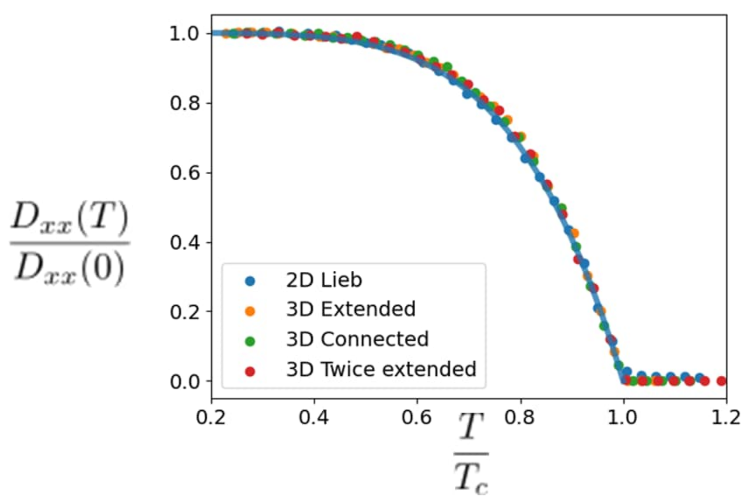
— $\sqrt{\det D(0)}\left(1-\left(\frac{T}{T_c}\right)^5\right)$

2D extensions





3D extensions



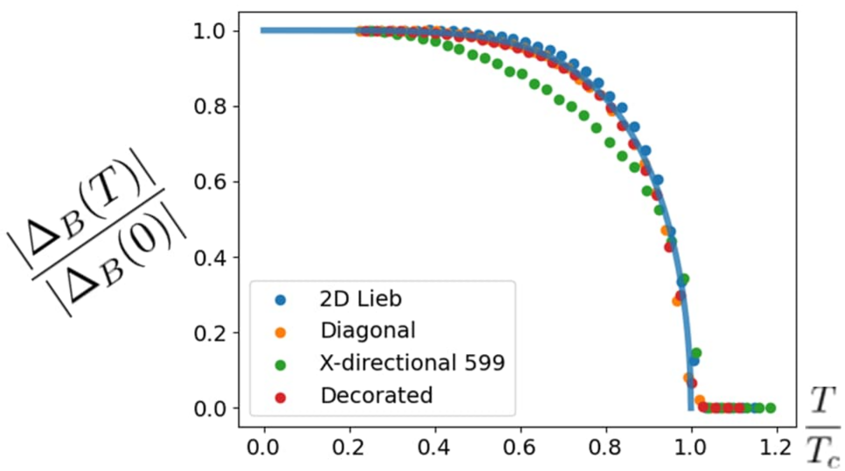
— $D_{xx}(0)\left(1-\left(\frac{T}{T_c}\right)^5\right)$

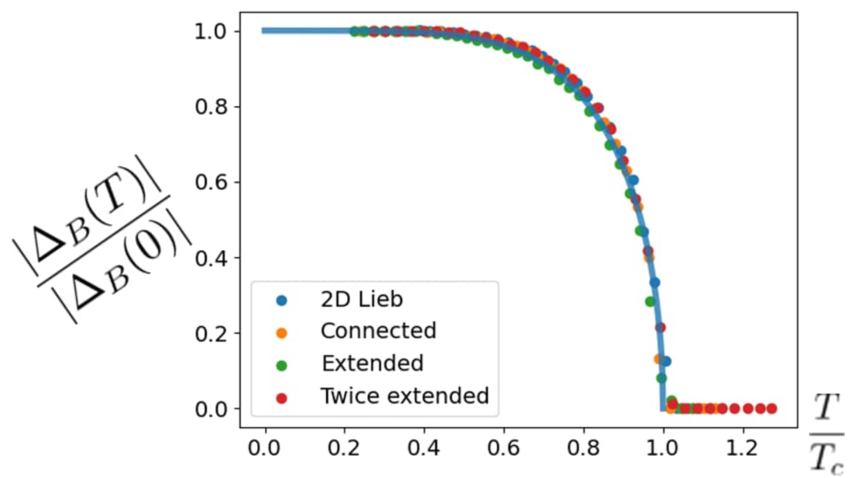
Qualitative behaviour of the order parameters

All lattices except X-directional extension have the same qualitative behavior

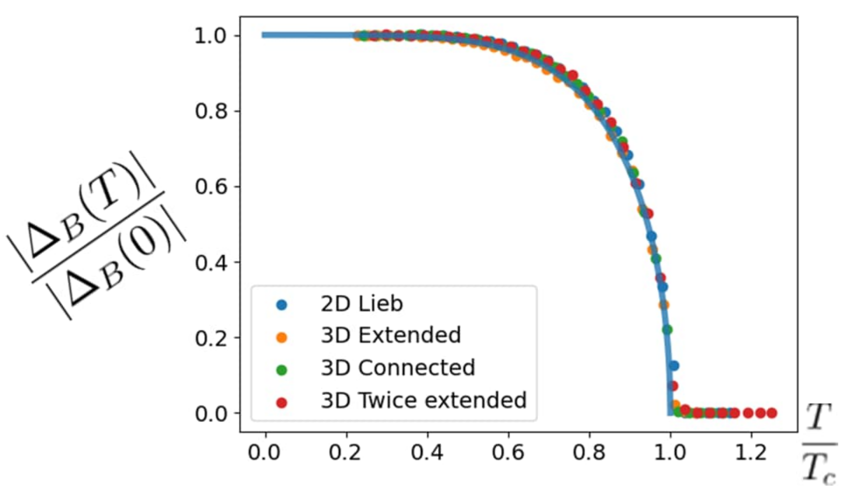
$$|\Delta_B(0)|\sqrt{1-\left(\frac{T}{T_c}\right)^5} \qquad \sqrt{\det D(0)}\left(1-\left(\frac{T}{T_c}\right)^5\right)$$

2D extensions



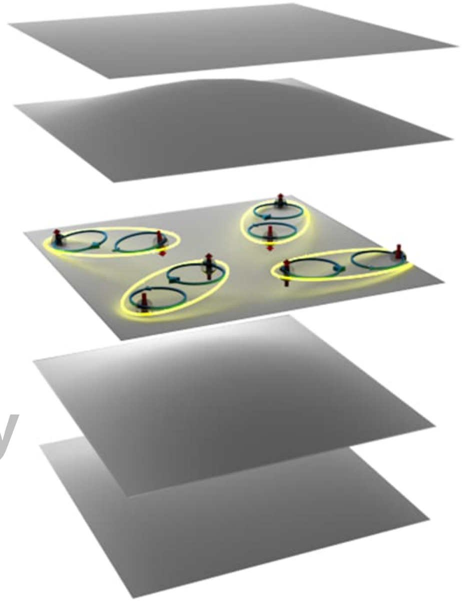


3D extensions



$$[D_s]_{ij} = \frac{2}{\pi\hbar^2}\frac{\Delta^2}{U N_{\rm orb}}\mathcal{M}^{\rm R}_{ij}$$

Contents



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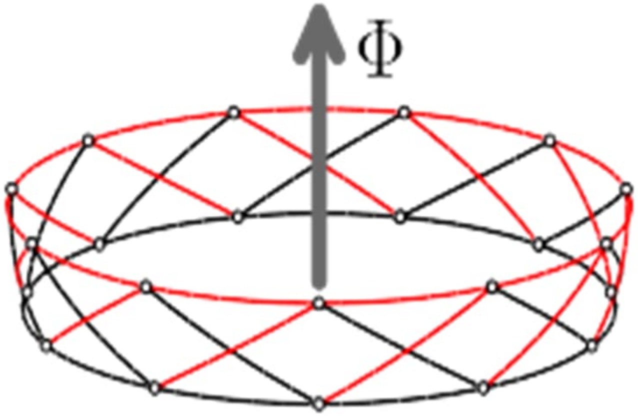
New phenomena also in the flat band normal state

In certain lattice models, only pairs move at any temperature, Tovmasyan, Peotta, Liang, PT, Huber, PRB 2018



Aharonov-Bohm effect in a ring geometry

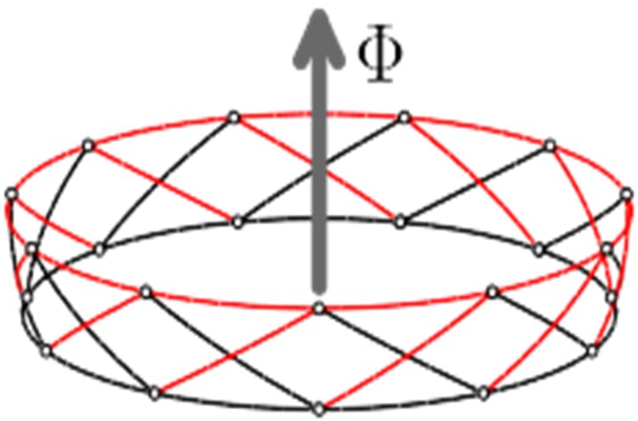
Non-Fermi liquid features in double occupancy and entropy (Lieb lattice), Kumar, Peotta, Takasu, Takahashi, PT, PRB(L) 2021

Insulator – pseudogap crossover in the Lieb lattice normal state, Huhtinen, PT, PRB(L) (2021)

Preformed pairs in a flat band

Tovmasyan, Peotta, Liang, PT, Huber, PRB 2018

What are the charge carriers in the ***normal state*** of a flat band superconductor?
We find: only pairs move (Pi-periodic ground state); non Landau-Fermi liquid.



Aharonov-Bohm effect in a ring geometry

Ground state energy vs. magnetic flux $E_0(\Phi)$

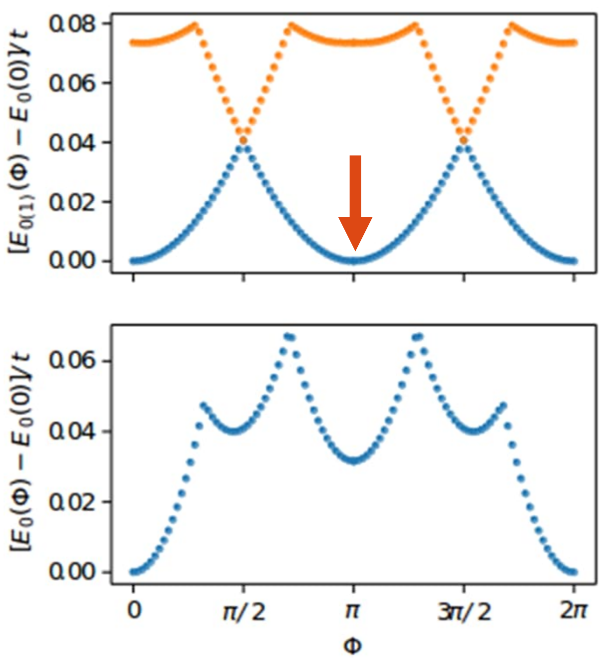
Flat band

$$E_0(\Phi) = E_0(\Phi + \Phi_0/2)$$

$$\Phi_0 = hc/e = 2\pi$$

$$\hbar = c = e = 1$$

Non Flat band



Related to local conserved quantities.

Flat band interacting normal state; Lieb lattice

- **Non-Fermi liquid features in double occupancy and entropy**
- **SU(N) scaling relation**

Pramod Kumar

Sebastiano Peotta

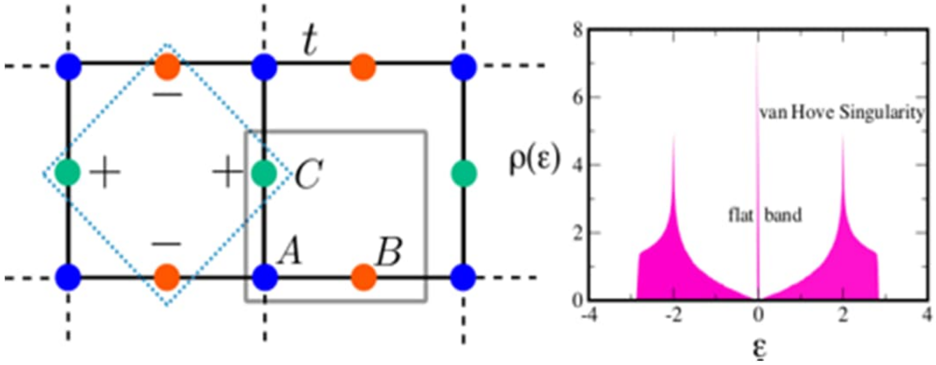
Yosuke Takasu

Yoshiro Takahashi

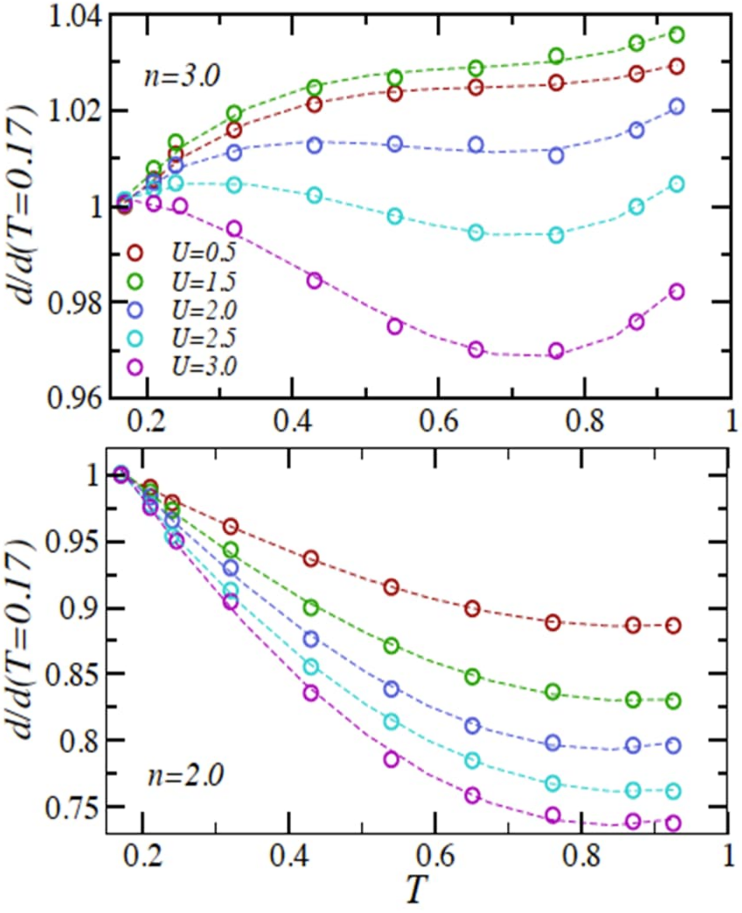
P Kumar, S Peotta, Y Takasu, Y Takahashi, PT, PRB(L) 2021

Lieb lattice: repulsive Hubbard model

Normal state properties



average double occupancy (DMFT)



half-filling: flat band significant

Non-Fermi liquid behavior for small interactions at the flat band

lowest band filled

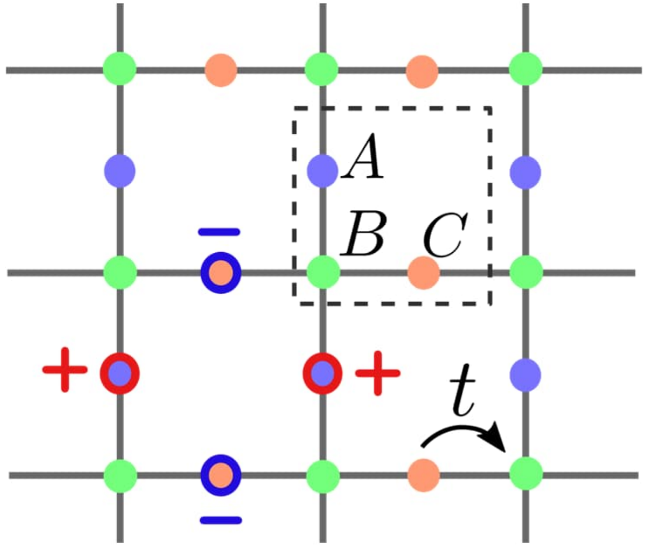
Insulator – pseudogap crossover in the Lieb lattice normal state

Kukka-Emilia Huhtinen

KE Huhtinen, PT, PRB(L) (2021)

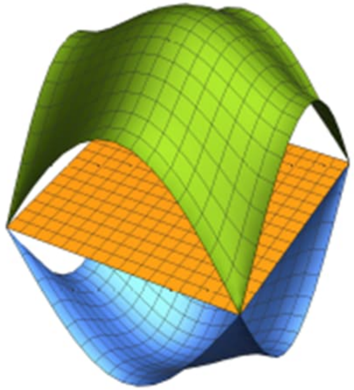
Hubbard model on the Lieb lattice

FOCUS ON THE NORMAL STATE ABOVE SUPERCONDUCTIVITY



Attractive Hubbard model

$$H = \sum_{\sigma} \sum_{i\alpha, j\beta} t_{ij} c^{\dagger}_{\sigma,i\alpha} c_{\sigma,j\beta} - \sum_{\sigma} \sum_{\mathrm{i}\alpha} \mu_{\sigma} n_{\sigma,i\alpha}$$
$$+U \sum_{i\alpha} (n_{\uparrow,i\alpha} - 1/2)(n_{\downarrow,i\alpha} - 1/2)$$



Flat band states reside at A and C sites

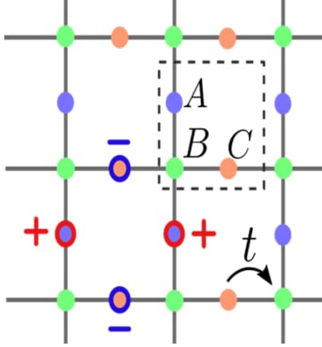
DMFT cluster: A, B and C

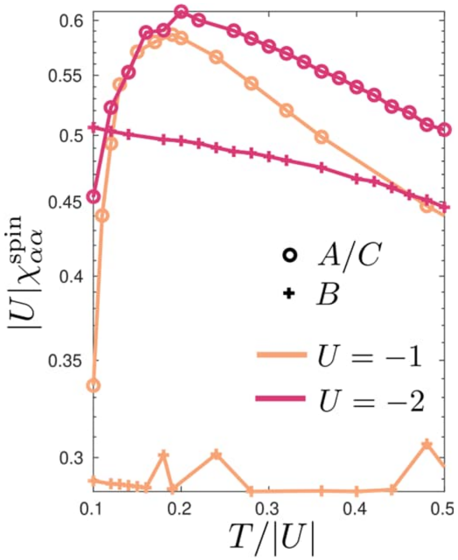
DMFT

Georges, Kotliar, Krauth, Rozenberg, Rev. Mod. Phys. 1996

Kotliar, Savrasov, Haule, Oudovenko, Parcollet, Marianetti, Rev. Mod. Phys. 2006

Large (U>t) interactions: pseudogap





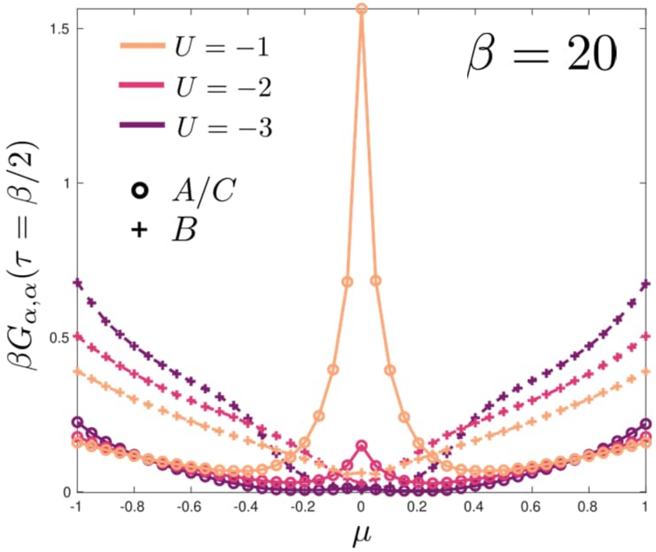
Generalized spin susceptibility:

$$\chi^{\rm spin}_{\alpha\alpha} = \frac{2}{\beta^2}\sum_{\omega,\omega'}\left(\chi^{{\rm ph},\omega,\omega',\nu=0}_{\uparrow\alpha,\uparrow\alpha,\uparrow\alpha,\uparrow\alpha} - \chi^{{\rm ph},\omega,\omega',\nu=0}_{\uparrow\alpha,\uparrow\alpha,\downarrow\alpha,\downarrow\alpha}\right)$$

$$\chi_{ijkl}(\tau_1,\tau_2,\tau_3) = G^{(4)}_{ijkl}(\tau_1,\tau_2,\tau_3) - G_{ij}(\tau_1,\tau_2)G_{kl}(\tau_3,0)$$

$$G^{(4)}_{ijkl}(\tau_1,\tau_2,\tau_3) = \langle T_\tau[c_i^\dagger(\tau_1)c_j(\tau_2)c_k^\dagger(\tau_3)c_l(0)]\rangle$$

$$G_{ij}(\tau_1,\tau_2) = \langle T_\tau[c_i^\dagger(\tau_1)c_j(\tau_2)]\rangle$$

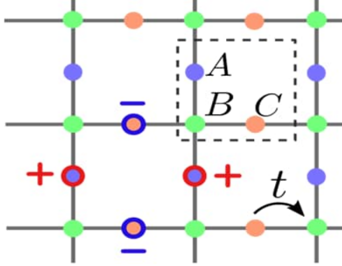


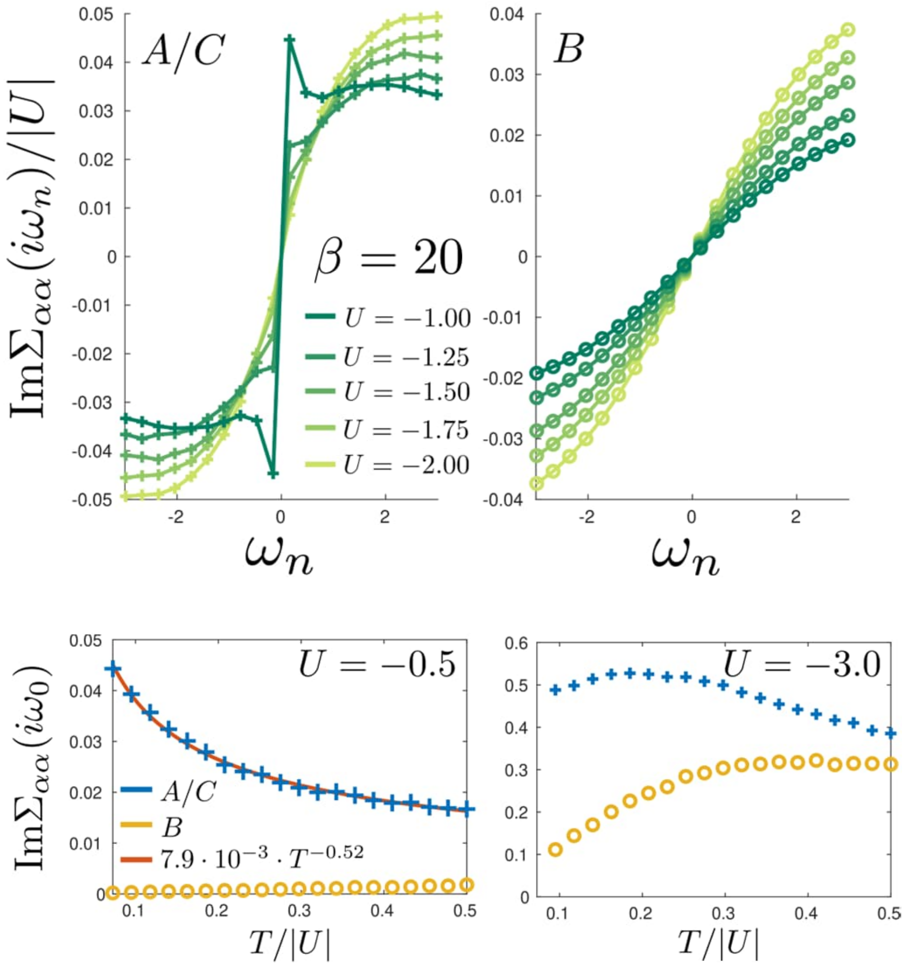
Local contribution to spin susceptibility decreases sharply with temperature at A/C sites.

At low temperatures, $\beta G_{\alpha\alpha}(\beta/2) \approx \mathcal{A}_\alpha(\omega=0)$, where $\mathcal{A}_\alpha$ is the orbital-resolved spectral function.

As interaction is increased, the spectral function becomes depleted around half-filling.

Low interaction (U<t): insulator





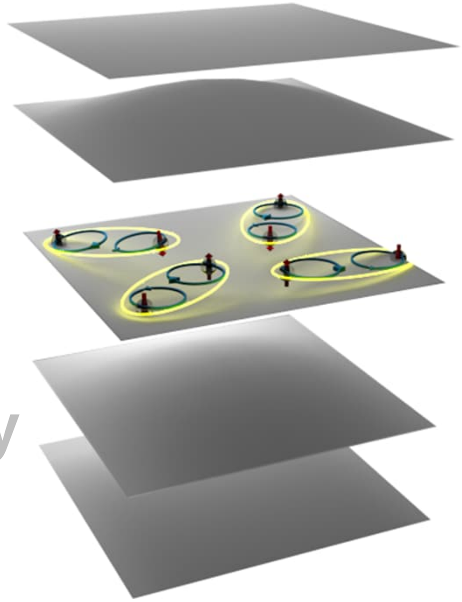
$$Z = \left(1 - \frac{\mathrm{Im}\Sigma(i\omega_n)}{\omega_n}\bigg|_{\omega_n \to 0}\right)^{-1}$$

In DMFT, $Z = m/m^*$, where m is the bare mass and m^* is the effective mass.

The self-energy diverges at low frequencies when the interaction strength is decreased.

The temperature dependence is $T^{-1/2}$ rather than T^{-1} found for Mott insulator.

Contents



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SYNOPSIS

PDF Version

Static Electrons in Flat-Band Nonequilibrium Superconductors

May 25, 2023 • *Physics* 16, s76

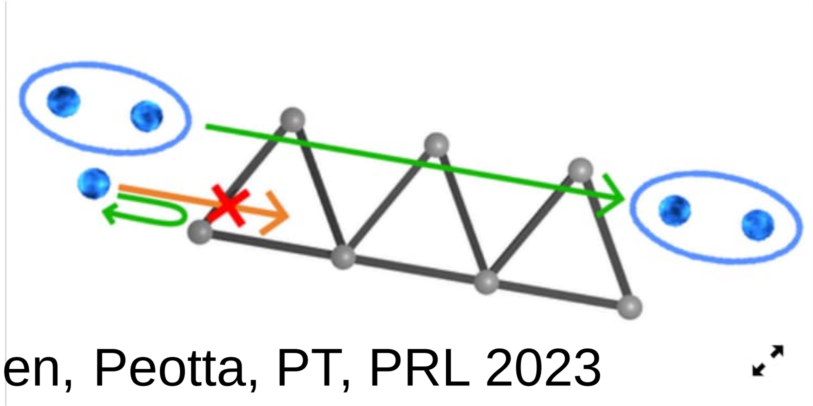
Single electrons stay stationary in superconductors with "flat-band" electronic structures, which could lead to low-energy-consumption devices made from such materials.

Suppression of Nonequilibrium Quasiparticle Transport in Flat-Band Superconductors

Ville A. J. Pyykkönen, Sebastiano Peotta, and Päivi Törmä

Phys. Rev. Lett. 130, 216003 (2023)

Published May 25, 2023



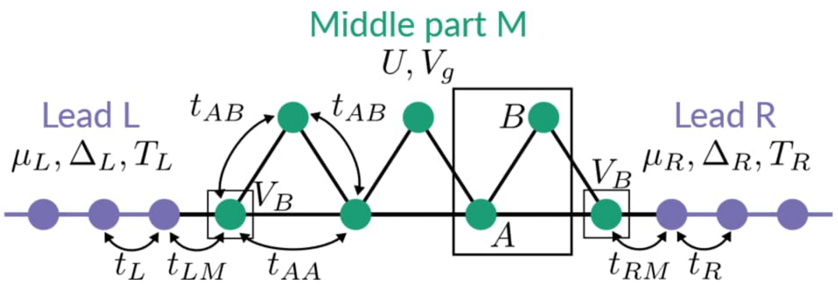
A. Paraoanu/Aalto University

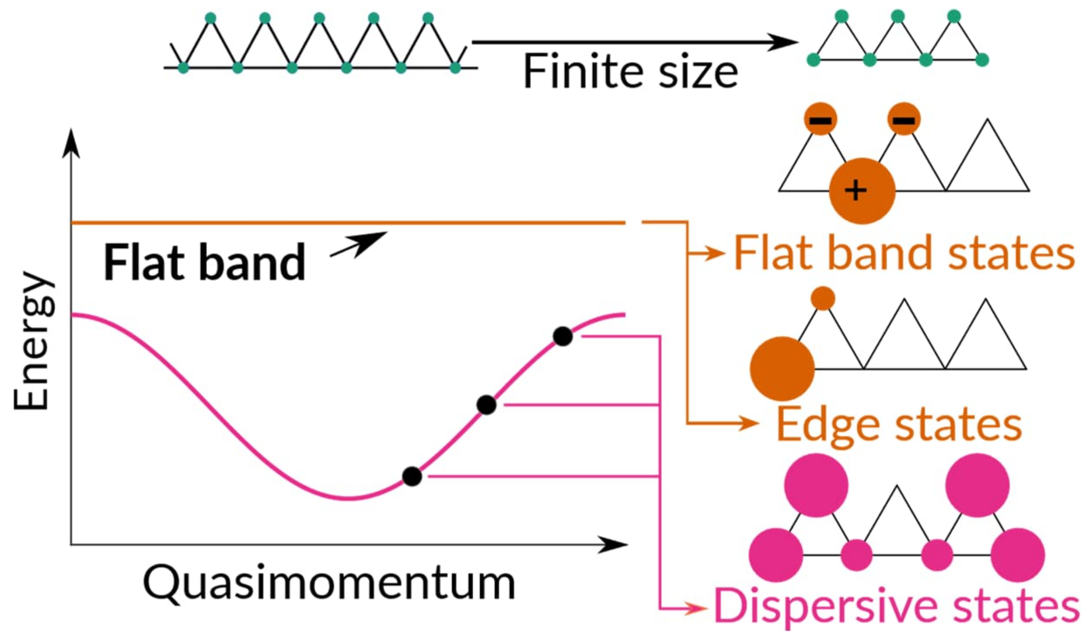
Pyykkönen, Peotta, PT, PRL 2023
Editors' Suggestion

Ville Pyykkönen

Sebastiano Peotta

Flat, edge and dispersive states in the sawtooth ladder

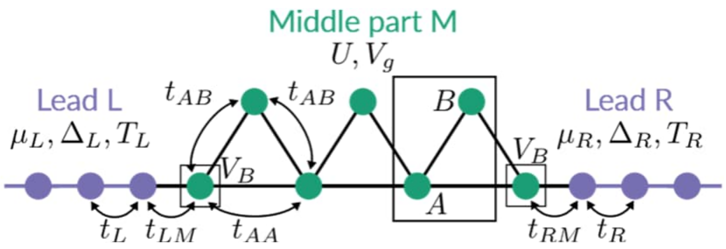




Select a state by gate potential

V_g

Flat band transport in Keldysh formalism



Fermi-Hubbard Hamiltonian

$$\hat{H} = \sum_{\alpha i,\beta j,\sigma} T_{\alpha i,\beta j}\hat{c}^\dagger_{\alpha i\sigma}\hat{c}_{\beta j\sigma} + \sum_{\alpha i} U_{\alpha i}\hat{c}^\dagger_{\alpha i\uparrow}\hat{c}^\dagger_{\alpha i\downarrow}\hat{c}_{\alpha i\downarrow}\hat{c}_{\alpha i\uparrow}$$

$$\alpha, \beta \in \{L, R, M\}$$

Mean-field approximation

$$\hat{H}_{\mathrm{MF}}(t) = \sum_{\alpha i,\beta j} \hat{d}^\dagger_{\alpha i} \begin{pmatrix} T_{\alpha i,\beta j} + V_{H,\alpha i}(t)\delta_{\alpha i,\beta j} & \Delta_{\alpha i}\delta_{\alpha i,\beta j} \\ \Delta^*_{\alpha i}\delta_{\alpha i,\beta j} & -T^*_{\alpha i,\beta j} - V_{H,\alpha i}(t)\delta_{\alpha i,\beta j} \end{pmatrix} \hat{d}_{\beta j}$$

$$\hat{d}_{\alpha i} = \left(\hat{c}_{\alpha i\uparrow}, \hat{c}^\dagger_{\alpha i\downarrow}\right)^{\mathrm{T}}$$

Superconducting order parameter

$$\Delta_{\alpha i}(t) = U_{\alpha i}\langle \hat{c}_{\alpha i\downarrow}(t)\hat{c}_{\alpha i\uparrow}(t)\rangle$$

Hartree potential

$$V_{H,\alpha i}(t) = U_{\alpha i}\langle \hat{c}^\dagger_{\alpha i\uparrow}(t)\hat{c}_{\alpha i\uparrow}(t)\rangle$$

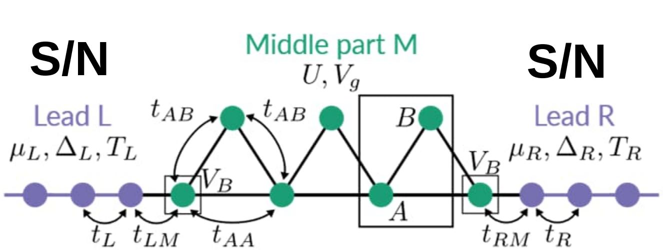
Keldysh formalism, non-equilibrium Green's functions

Dyson equation $G^{R/A}(\omega) = g^{R/A}(\omega) + g^{R/A}(\omega)\Sigma^{R/A}(\omega)G^{R/A}(\omega)$

Kadanoff-Baym kinetic equation

$$G^<(\omega) = \left[I + G^R(\omega)\Sigma^R(\omega)\right] g^<(\omega) \left[I + \Sigma^A(\omega)G^A(\omega)\right] + G^R(\omega)\Sigma^<(\omega)G^A(\omega)$$

Transport



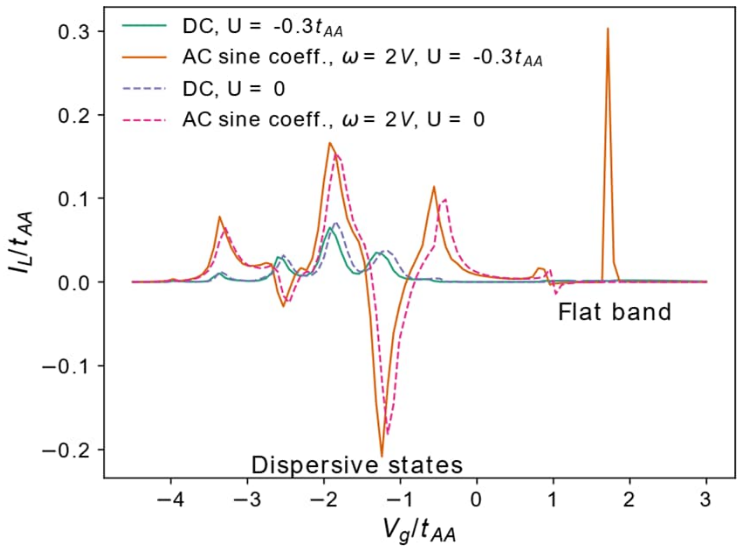
Constant bias

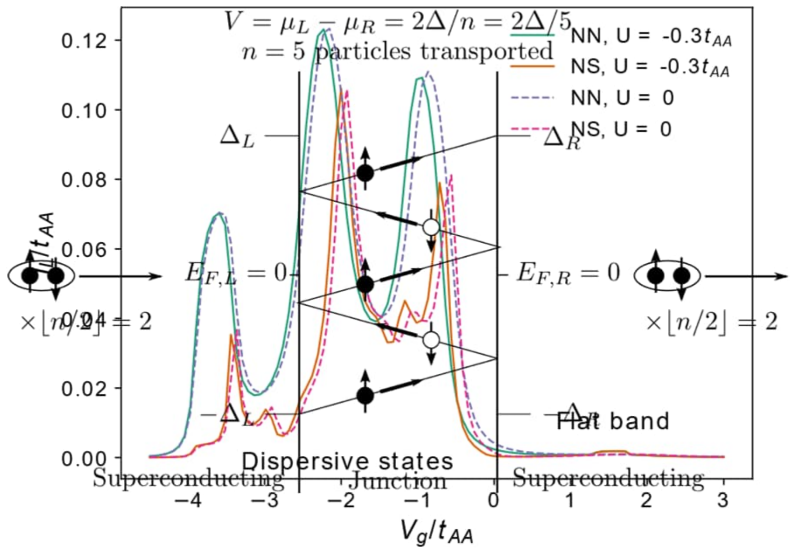
$$V = \mu_L - \mu_R$$

Vary gate potential

$$V_g$$

Current $$I_L(t) = 4\mathrm{Im}\left(t_{ML}\left\langle \hat{c}^\dagger_{M,\text{left edge},\uparrow}(t)\hat{c}_{L,0\uparrow}(t)\right\rangle\right)$$





Superconducting junction: at finite interaction **flat band AC Josephson current is finite** but **DC current (multiple Andreev reflections) quenched**

Normal-normal and normal-superconducting junction: **flat band current is quenched**

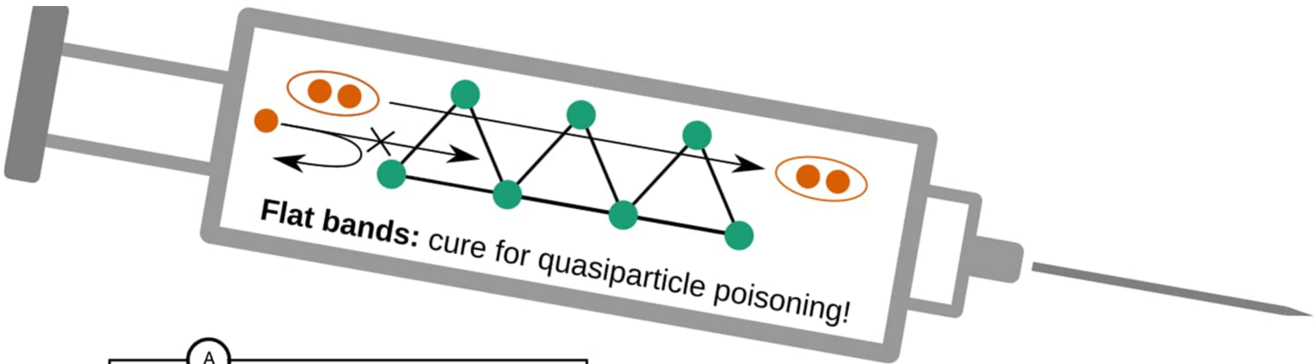
Quasiparticle transport quenched at flat band! Pure supercurrent!

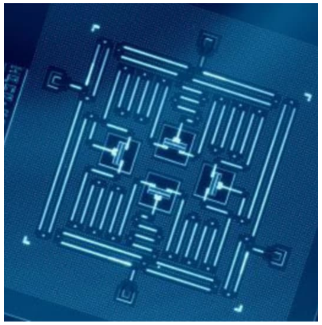
Quasiparticle transport quenched at flat band! Pure supercurrent!

Quasiparticle poisoning

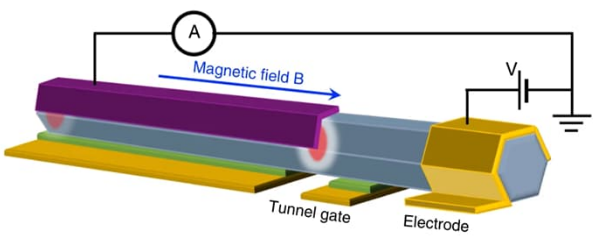
Nonequilibrium Quasiparticles and $2e$ Periodicity in Single-Cooper-Pair Transistors

J. Aumentado, Mark W. Keller, John M. Martinis, and M. H. Devoret
Phys. Rev. Lett. **92**, 066802 – Published 13 February 2004





Four transmons. FJ.M. Gambetta, J.M. Chow, and M. Steffen (npj QuantumInformation 3:2, 2017) by CC BY 4.0 license



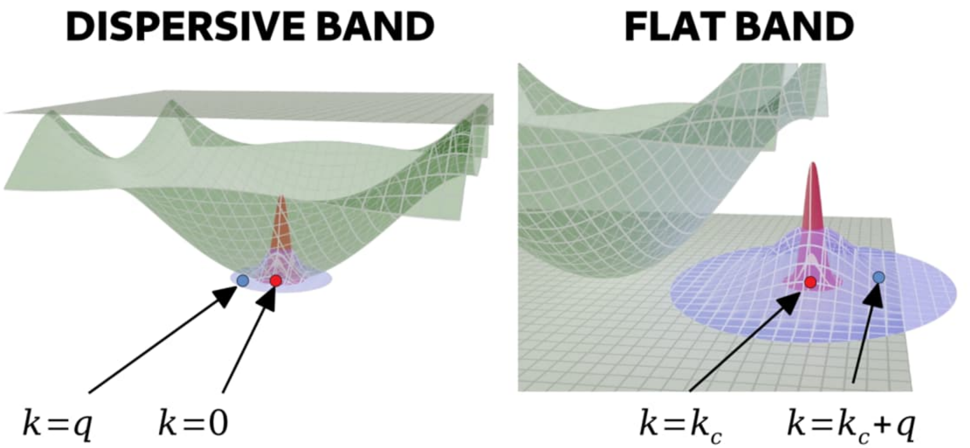
Majorana nanowire. H. Zhang, D.E. Liu, M. Wimmer, L.P. Kouwenhoven (Nat Commun 10, 5128, 2019) by CC BY 4.0 license

G. Catelani and J. P. Pekola, Using materials for quasiparticle engineering, Materials for Quantum Technology 2, 013001 (2022)

D. Rainis and D. Loss, Majorana qubit decoherence by quasiparticle poisoning, Phys. Rev. B 85, 174533 (2012)

Flat band BEC & quantum geometry

Kagome lattice:



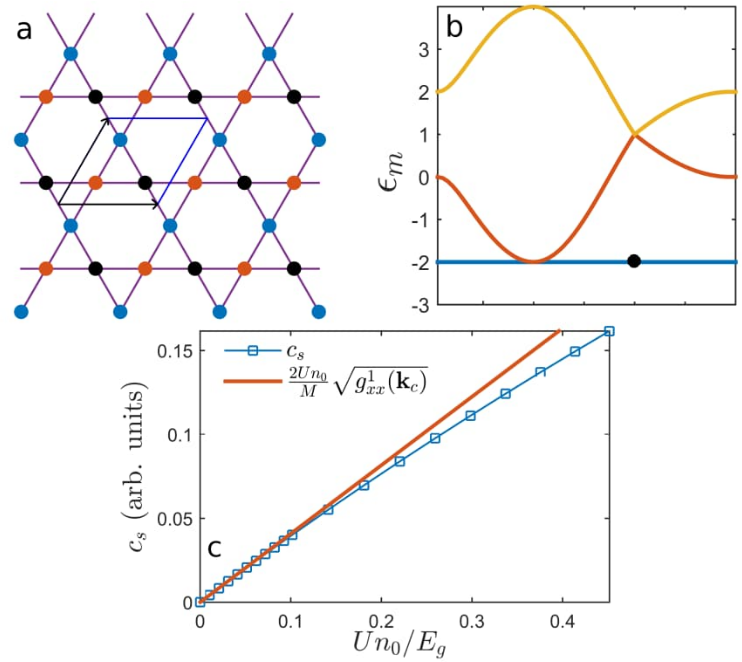
n_0 Condensate density n_e Excitation density U Interaction $u(k)$ Bloch function

SPEED OF SOUND

$c_s \propto \sqrt{U} n_0$ $\qquad$ $c_s \propto U n_0 \sqrt{g_{\alpha\beta}(k_c)}$

Quantum metric

$$g_{\alpha\beta} = \Re[\langle \partial_\alpha u | \partial_\beta u \rangle - \langle \partial_\alpha u | u \rangle \langle u | \partial_\beta u \rangle]$$



Quantum metric dictates the speed of sound

Aleksi Julku

Georg Bruun

Grazia Salerno

Julku, Bruun, PT, PRL 2021, PRB 2021
Julku, Salerno, PT, Fizika Nizkikh Temperatur (journal of ILTPE, Kharkiv, Ukraine) 49, 770 (2023); special issue coordinated by Andrei Bernevig, Princeton

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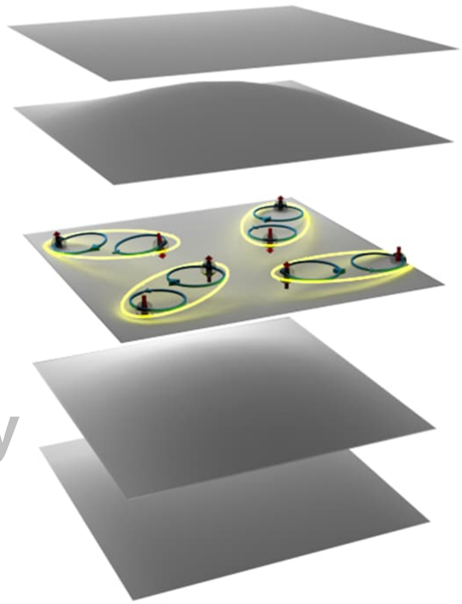

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Conductivity in a flat band

Kukka-Emilia Huhtinen

KE Huhtinen, PT, PRB (2023)

Conductivity in a flat band

Semiclassical Boltzmann theory of transport:

$$\sigma_{\mu\nu}(\omega) = -\frac{e^2}{\hbar}\sum_n \int_{\text{B.z.}} \frac{d^D\boldsymbol{k}}{(2\pi)^D} \frac{\partial n_F(E)}{\partial E}\bigg|_{E=\epsilon_n(\boldsymbol{k})} \partial_\mu \epsilon_n(\boldsymbol{k}) \partial_\nu \epsilon_n(\boldsymbol{k}) \frac{\eta}{(\hbar\omega)^2+\eta^2} \qquad \partial_\mu = \partial/\partial k_\mu$$

Full Kubo-Greenwood formula:

$$\sigma_{\mu\nu}(\omega) = \frac{e^2}{i\hbar V}\sum_{\boldsymbol{k}}\sum_{mn} \frac{n_F(\epsilon_n(\boldsymbol{k})) - n_F(\epsilon_m(\boldsymbol{k}))}{\epsilon_n(\boldsymbol{k}) - \epsilon_m(\boldsymbol{k})} \frac{[j_\mu(\boldsymbol{k})]_{nm}[j_\nu(\boldsymbol{k})]_{mn}}{\epsilon_n(\boldsymbol{k}) - \epsilon_m(\boldsymbol{k}) + \hbar\omega + i\eta}$$

$$[j_\mu(\boldsymbol{k})]_{mn} = \partial_\mu \epsilon_m(\boldsymbol{k})\delta_{mn} + (\epsilon_m(\boldsymbol{k}) - \epsilon_n(\boldsymbol{k}))\langle \partial_\mu m_{\boldsymbol{k}} | n_{\boldsymbol{k}} \rangle$$

At low temperatures and finite scattering rate η , the interband geometric part is dominant on a flat band.

Insprired by

G. Bouzerar and D. Mayou, Phys. Rev. B 103, 075415 (2021)
J. Mitscherling and T. Holder, Phys. Rev. B 105, 08515 (2022)
B. Mera and J. Mitscherling, Phys. Rev. B 106, 165133 (2022)
G. Bouzerar, Phys. Rev. B 106, 125125 (2022)

Conductivity in a flat band

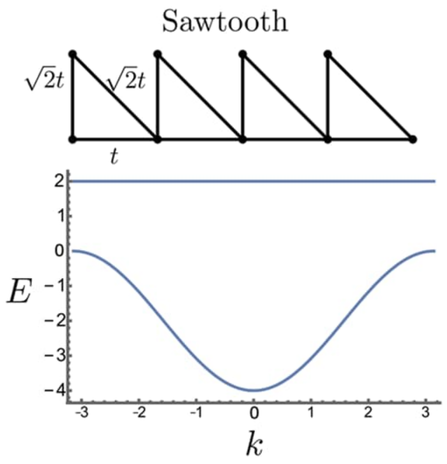
Streda formula:

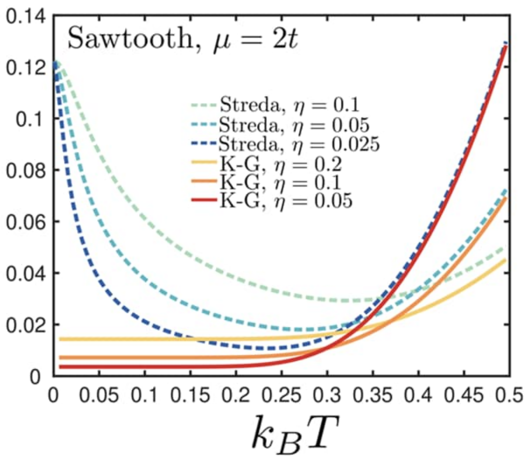
$$\sigma_{\mu\nu}^{\rm sym}(\omega=0) = -\frac{e^2}{\hbar\pi}\int_{-\infty}^{\infty} {\rm d}\epsilon \frac{\partial n_F(\epsilon)}{\partial\epsilon}{\rm Tr}[{\rm Im}[G_{\boldsymbol{k}}(\epsilon+i\eta)]j_\mu(\boldsymbol{k}){\rm Im}[G_{\boldsymbol{k}}(\epsilon+i\eta)]j_\nu(\boldsymbol{k})]$$

$$G_{\boldsymbol{k}}(E) = (E - H_{\boldsymbol{k}})^{-1}$$

This gives a result proportional to the integrated quantum metric in the limit $\eta \to 0^+$ when $T \to 0$ is taken *first.*

This occurs only in *perfectly* (partially) flat bands due to ill-defined terms for states at the Fermi energy. **The Kubo-Greenwood and Streda formulas do not give the same conductivity when a flat band is in the vicinity of the Fermi energy.**





Lack of Fermi surface requires extra care in transport calculations.

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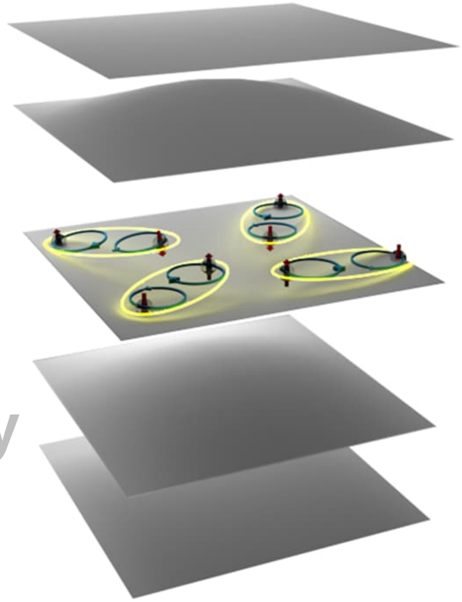

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Drude weight and the many-body quantum metric

Grazia Salerno

Tomoki Ozawa

Salerno, Ozawa, PT, PRB Letter (2023)

The many-body quantum metric (MBQM)

Defined on many-body states with respect to the twisted boundary condition phase

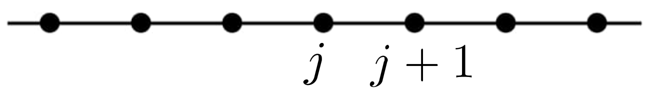
$$\mathfrak{g}(\phi) = \mathrm{Re}\left[\langle \partial_\phi \Psi_0 | \left(1 - |\Psi_0\rangle\langle\Psi_0|\right) |\partial_\phi \Psi_0\rangle\right]$$

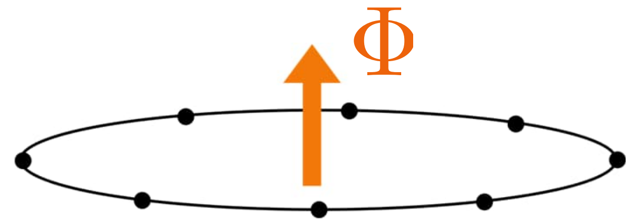
determines the ‘‘quantum distance’’ along a given path in ϕ space.

→ Many-body generalization of the quantum metric

$$\mathfrak{g}(0) = \mathrm{Re}\left[\sum_{m \neq 0} \frac{|\langle \Psi_m | \partial_\phi \hat{H}(\phi) | \Psi_0 \rangle|^2}{(E_m(0) - E_0(0))^2}\right]$$

Drude weight and twisted boundary conditions





$$\hat{H}_0 = \hat{H}_{\text{kin}} + \hat{H}_V + \hat{H}_U = (\hat{K} + \hat{K}^\dagger) + \hat{H}_V + \hat{H}_U$$

Superfluid response of the system to a small external flux Φ introduced by the twisted boundary conditions:

$$D_w = \pi L \left. \frac{\partial^2 E(\Phi)}{\partial \Phi^2} \right|_{\Phi=0}$$

$$\hat{H}(\Phi) = \hat{K} e^{i\Phi/L} + \hat{K}^\dagger e^{-i\Phi/L} + \hat{H}_V + \hat{H}_U$$

Drude weight within perturbation theory

$$\hat{H}(\Phi) = \hat{H}_0 + \hat{H}_{\text{pert}} \qquad \text{with} \qquad \hat{H}_{\text{pert}} = \frac{\Phi}{L}\hat{J} - \frac{1}{2}\left(\frac{\Phi}{L}\right)^2 \hat{H}_{\text{kin}}$$

Current operator $\hat{J} = i(\hat{K} - \hat{K}^\dagger)$

$$D_w = 2\pi L \frac{E(\Phi) - E(0)}{\Phi^2} = -\frac{\pi}{L}\langle\Psi_0|\hat{H}_{\text{kin}}|\Psi_0\rangle - \underbrace{\frac{2\pi}{L}\sum_{m\neq 0}\frac{|\langle\Psi_m|\hat{J}|\Psi_0\rangle|^2}{E_m(0) - E_0(0)}}$$

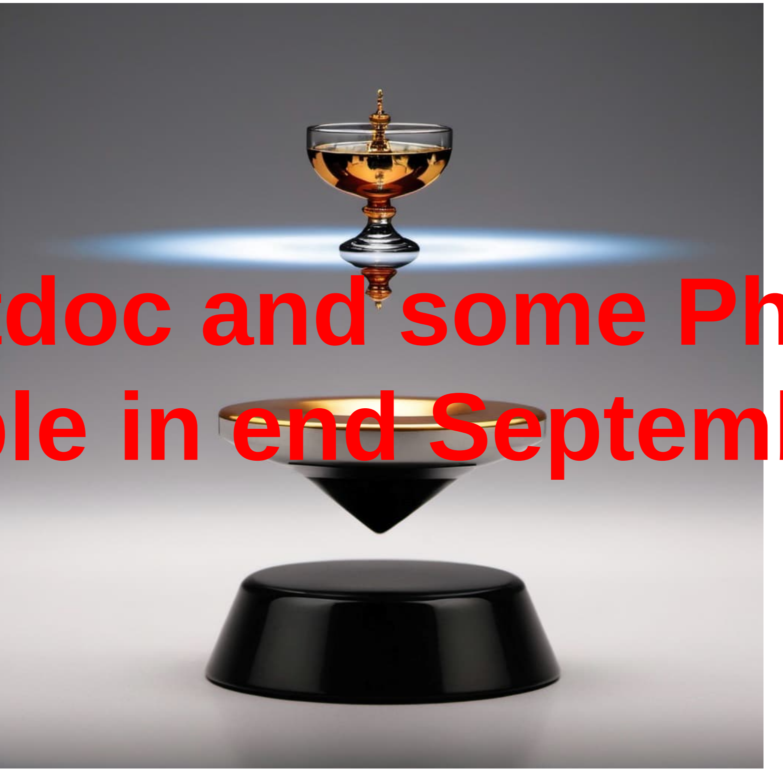
$> \mathfrak{g}(0)\cdot\varepsilon$

Can be bounded by the **many-body quantum metric**
if the system has a gap ε

Independent of particle statistics and spatial dimensions!

Simons Collaboration on New Frontiers in Superconductivity 2024-2028(2031)

SIS



Many postdoc and some PhD positions available in end September 2024

SIMONS FOUNDATION

SIS

Simons Collaboration on New Frontiers in Superconductivity

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Weizmann

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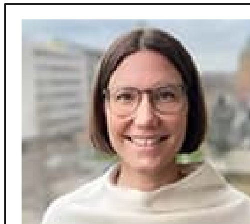
Minnesota



Laura Classen
Max Planck I

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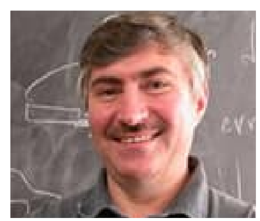
Postdoc positions available in many of these groups in end September 2024

Aline Ramires
Paul Scherrer I

Jonathan Ruhman Bar Ilan

Jörg Schmalian
KIT



Boris Svistunov
Massachusetts

Päivi Törmä
Aalto

Oskar Vafek
Florida State

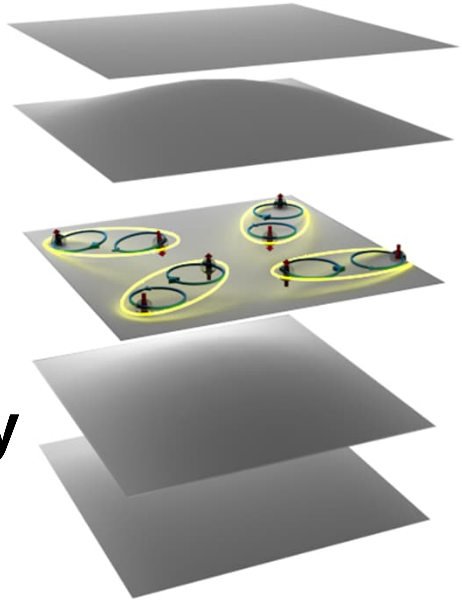
If interested in my group, you may discuss with me here

Simons Collaboration on New Frontiers in Superconductivity

Unsupervised AI

Theory by humans

Summary



Lecture 1

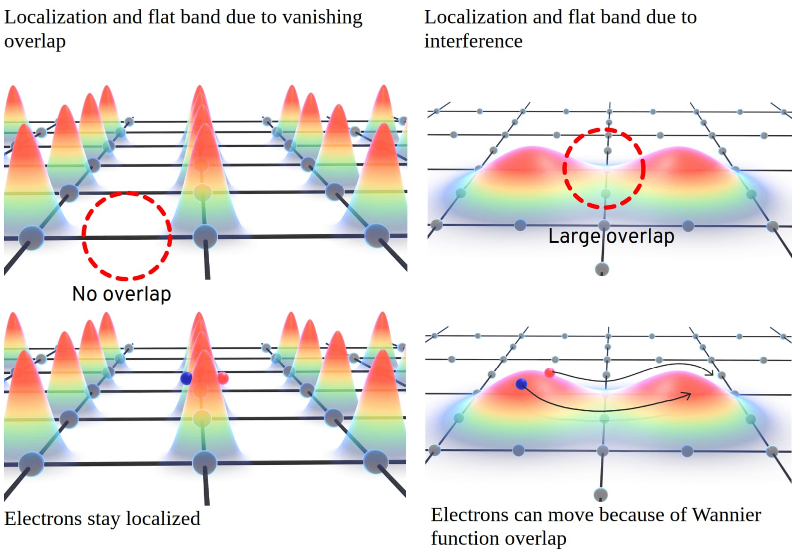
- Basics of quantum geometry
- Quantum geometry and superconductivity

Lecture 2

- Flat band superconductivity and quantum geometry in twisted bilayer graphene (TBG)
- Non-Fermi liquid normal states in flat bands
- Non-equilibrium transport in flat band superconductors
- DC conductivity in a flat band
- The many-body quantum metric and the Drude weight

Summary

Quantum geometry is relevant for any transport or interaction phenomena where overlaps and localization properties of Wannier functions are important – a new viewpoint to condensed matter physics: not only the band structure, but the structure of the Bloch functions



Outlook

Superconductivity at elevated temperatures

Aalto University
School of Science

Super C
Room Temperature
Superconductivity 2033

Institute Q

erc

HORIZON2020
Marie Skłodowska-
Curie actions

ACADEMY
OF FINLAND