Solvable Models for Strange Metals

Erez Berg

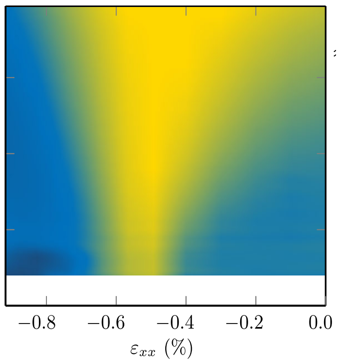
Weizmann Institute of Science

CRC 183 ENTANGLED STATES OF MATTER

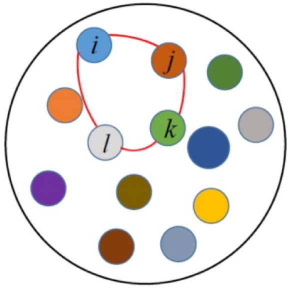
erc European Research Council

Outline

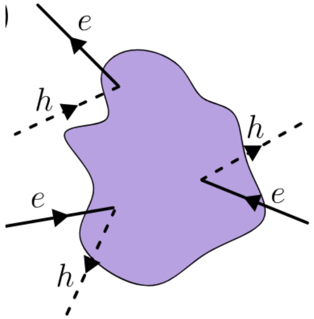
- What is a strange metal?



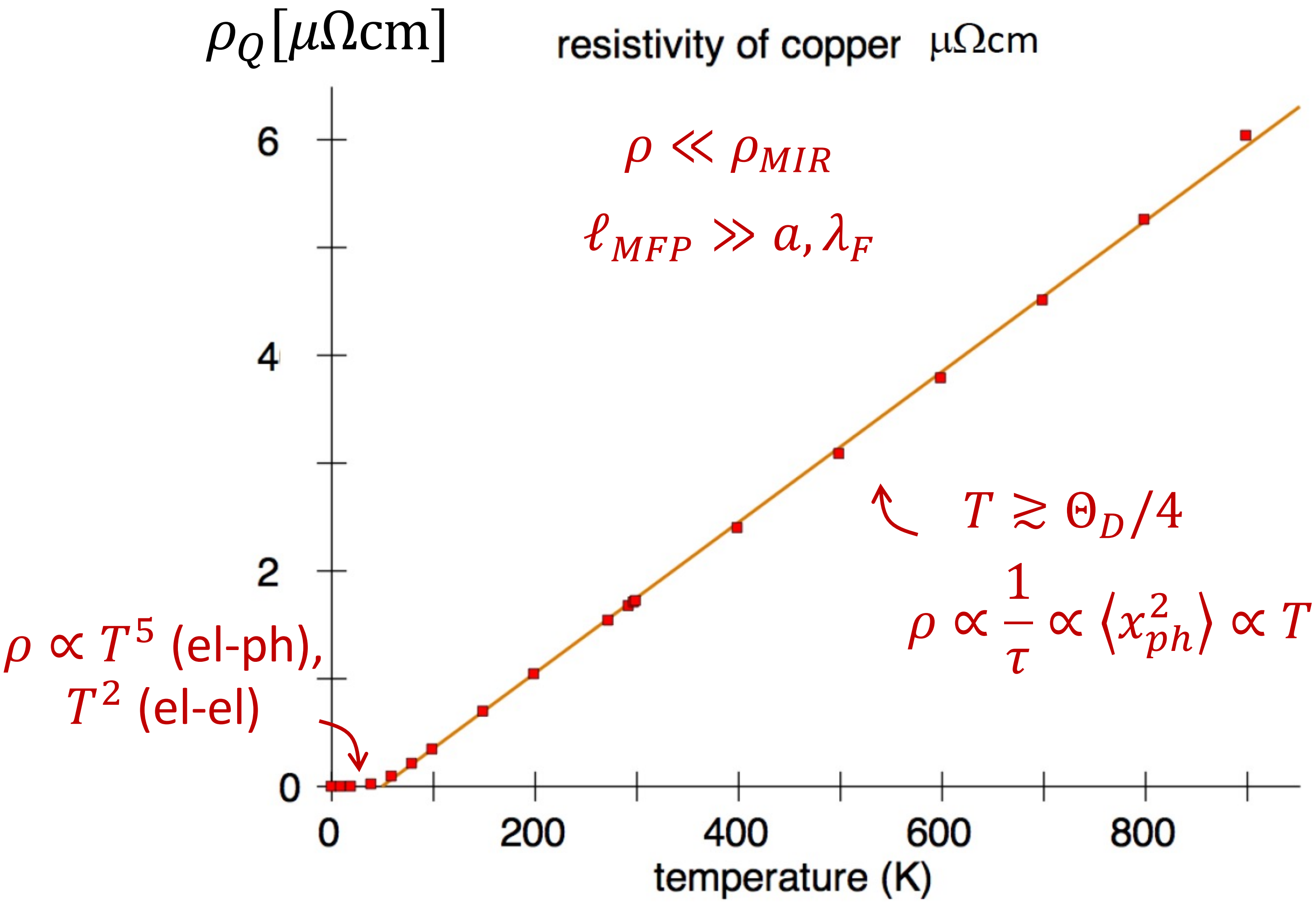
- A view from the large-N limit



- Strange metal from superconducting puddles



Resistivity of a conventional metal



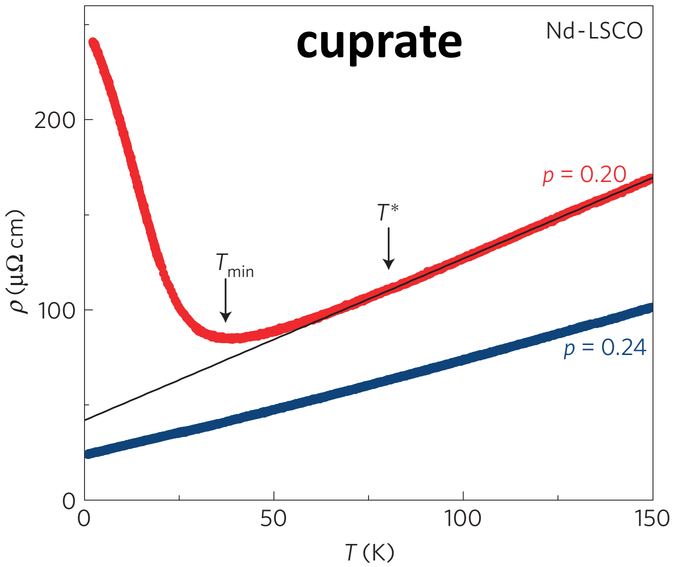
What is a strange metal?

A metal that cannot be explained by the conventional paradigm (Fermi liquid)+Boltzmann theory

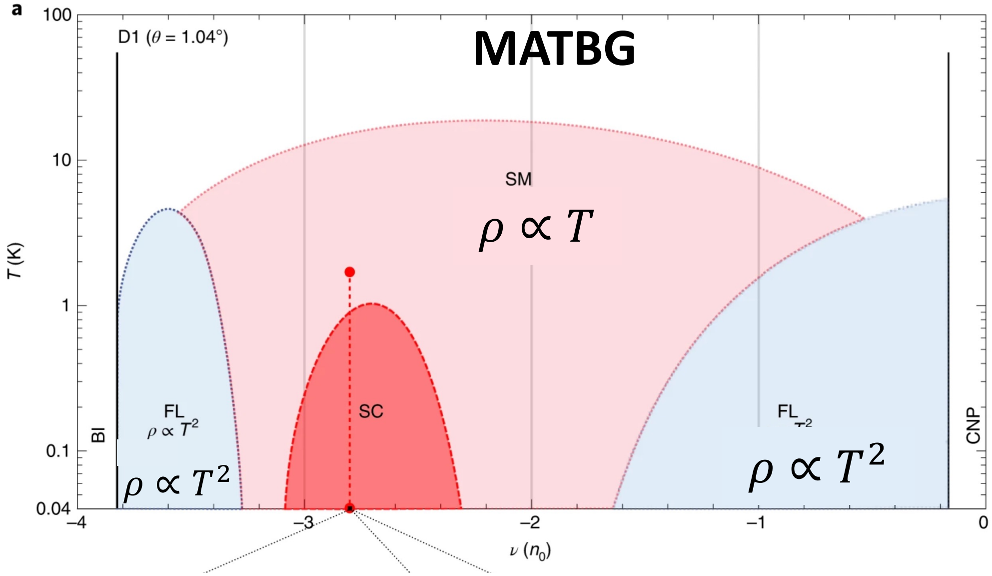
In the limit $T \to 0$.

Common diagnostic: the resistivity $\rho(T) = \rho_0 + AT^\alpha$

With $\alpha < 2$ (often $\alpha = 1$)



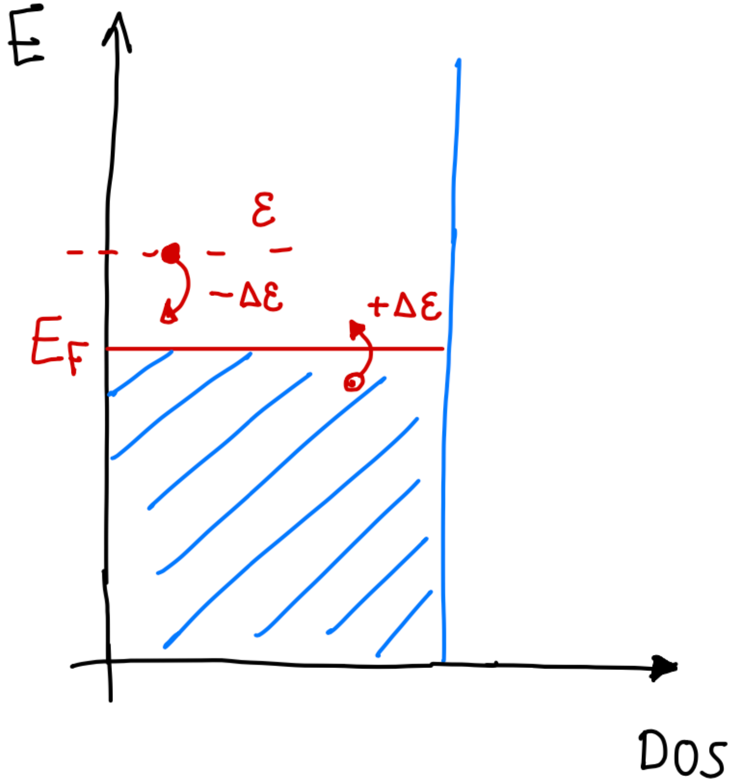
Daou,…,Taillefer *(2009)*

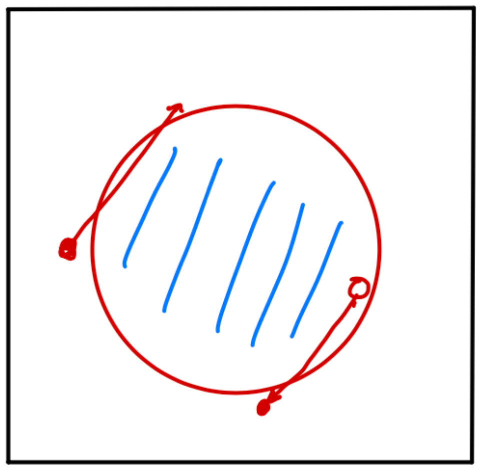


Jaoui,…,Efetov *(2022)*

What is a strange metal?

In a Fermi liquid, $\frac{1}{\tau} \propto \max(T^2, \varepsilon^2)$

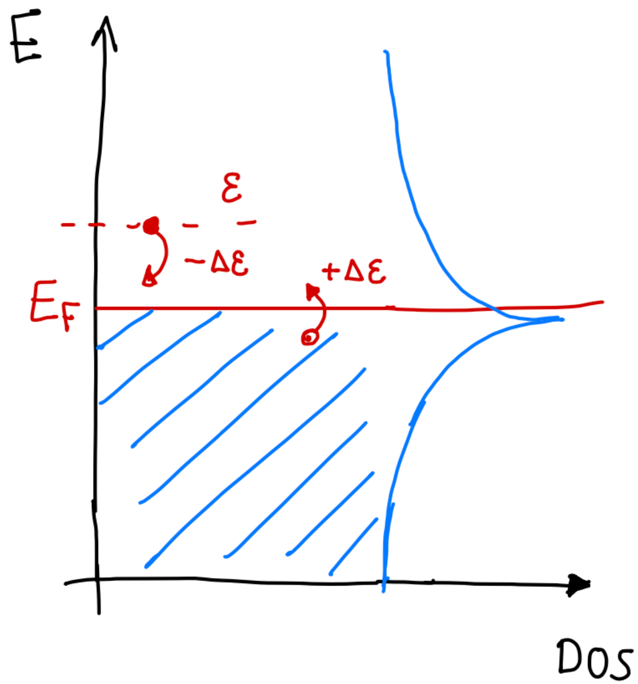




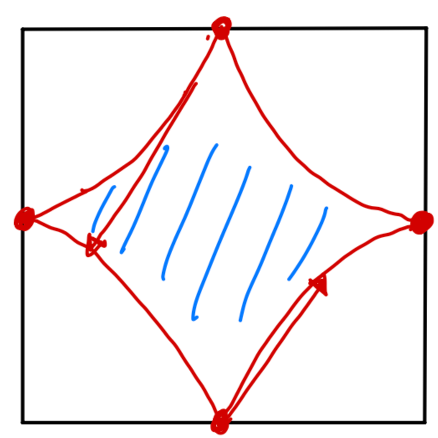
Strange metal

$\frac{1}{\tau} \propto \max(T^{\alpha}, \varepsilon^{\alpha})$ requires high DOS of low-energy excitations

VHS in 2D with $\nu(\varepsilon) \propto \nu_0 \log\frac{W}{\varepsilon}$ gives $\rho(T) \propto T^2 \ln\frac{W}{T}$



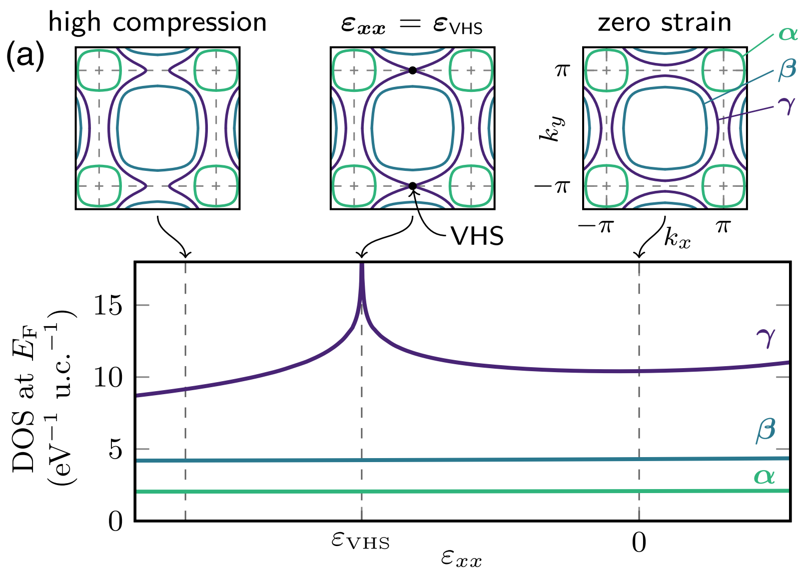
singular DOS
(e.g. 2D VHS)



Strain Turning through VHS

Sr_2RuO_4

Barber, Hicks et al. (2018)

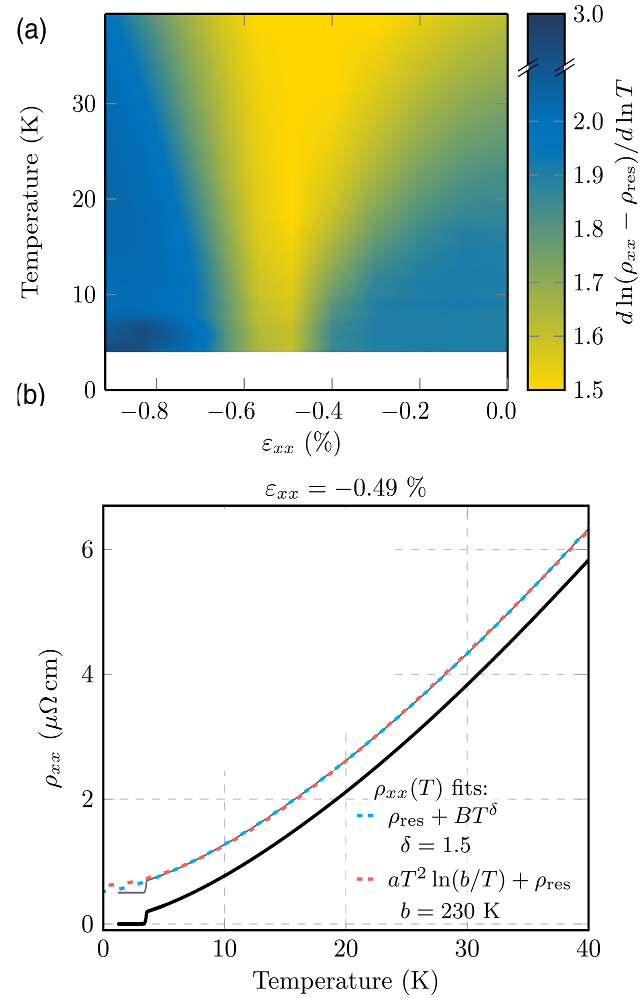


Resistivity at ε_{VHS} consistent with

$\rho \sim T^2 \log(1/T)$

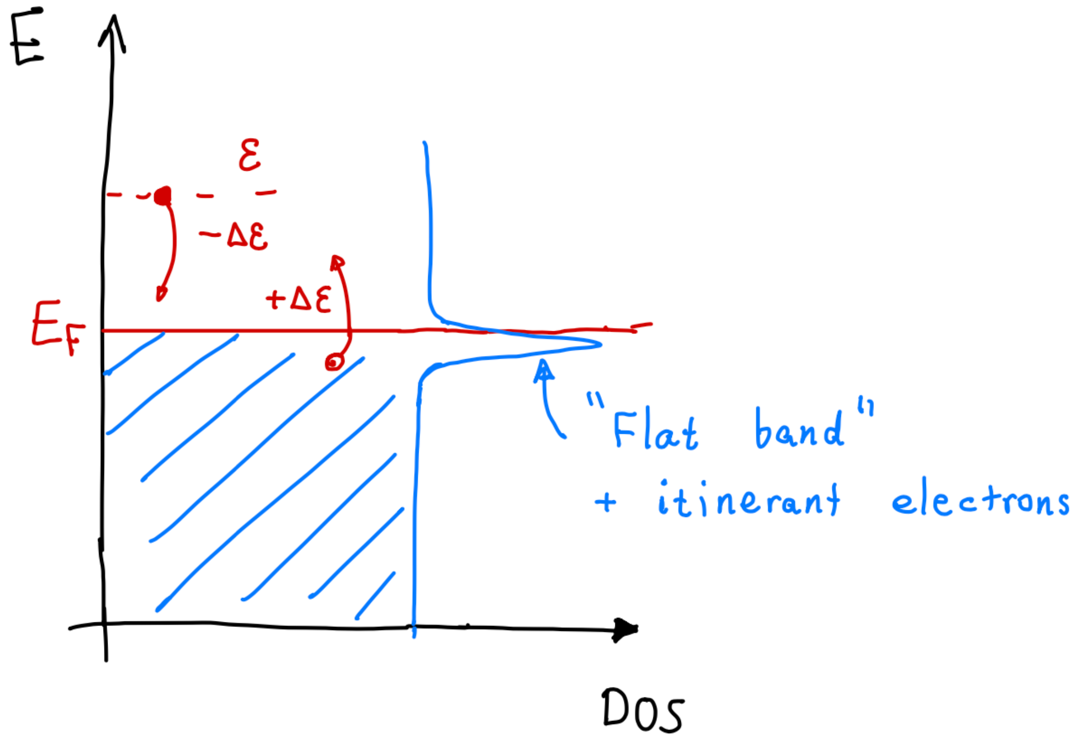
Mousatov, EB, Hartnoll (2020);

Stangier, EB, Schmalian (2022)



Strange metal from flat+itinerant bands?

What DOS gives $\rho - \rho_0 \propto T$?

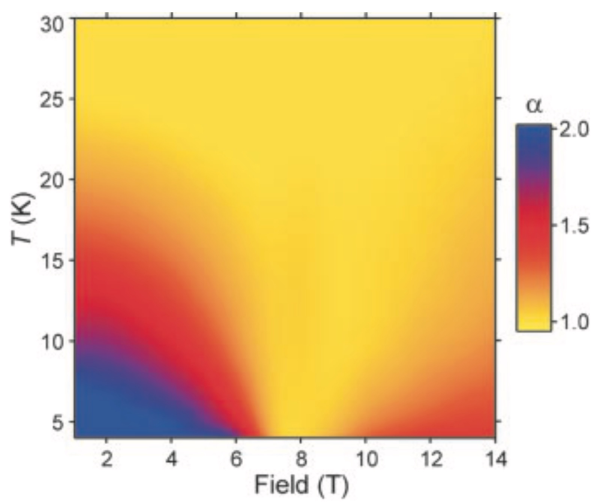


Mousatov, EB, Hartnoll (2020)

Quantum Critical Point?

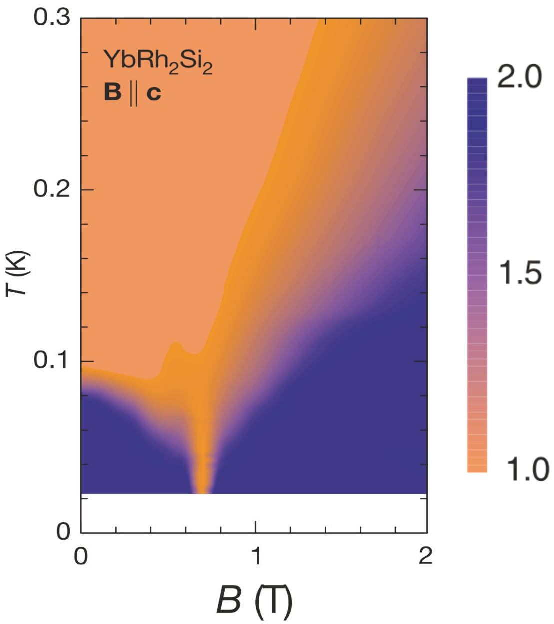
$$\rho \propto T, \frac{c}{T} \propto \log\frac{1}{T} \text{ in metals tuned to a QCP}$$

$\alpha = d\ln(\rho - \rho_0)/d\ln T$



$Sr_3Ru_2O_7$

Grigera,…, Mackenzie (02')



$YbRh_2Si_2$

Custers,…, Steglich, Pepin, Coleman (03')

Outline

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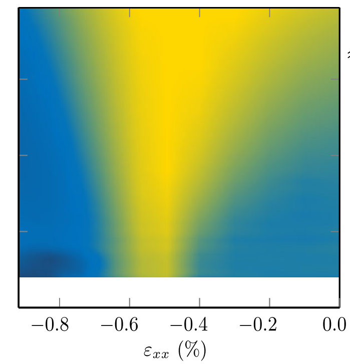

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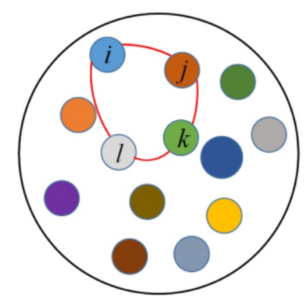
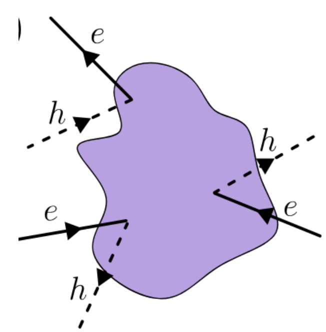
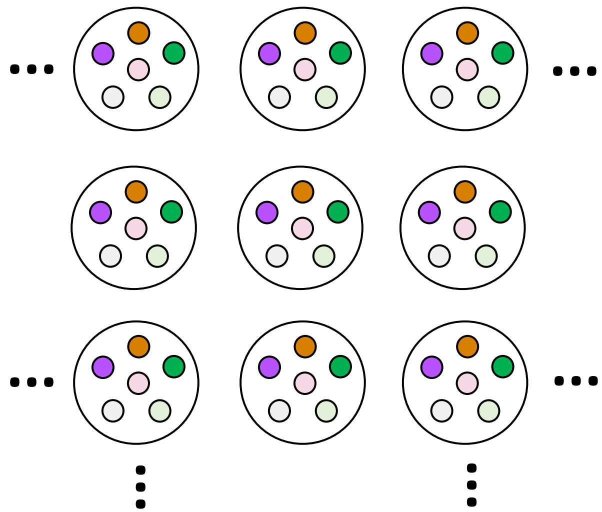


- Strange metal from superconducting puddles



“Standard models” of strongly correlated electrons

N-band Hubbard models:



$$H = \sum_{i,j} \sum_{\alpha,\beta=1}^{N} t_{i,j}^{\alpha,\beta} c_{i,\alpha}^{\dagger} c_{j,\beta} + \sum_{i,j,k,l} \sum_{\alpha,\beta,\gamma,\delta} U_{ijkl}^{\alpha\beta\gamma\delta} c_{i,\alpha}^{\dagger} c_{j,\beta}^{\dagger} c_{k,\gamma} c_{l,\delta}$$

$+$ disorder (lattice defects)

$+$ phonons (lattice vibrations)

$+ \cdots$

i: lattice site

$\alpha = 1 \ldots N$: orbital index

"Standard models" of strongly correlated electrons

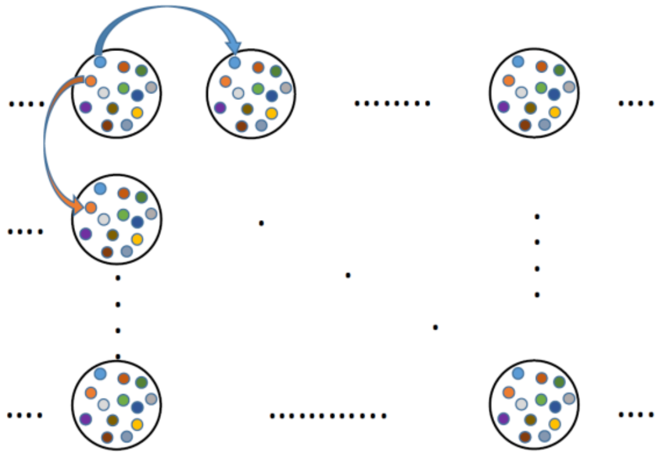
$N \to \infty$ limit

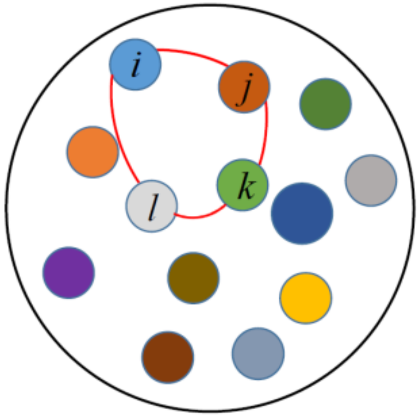
- Introduce a symmetry (exact or statistical) between the orbitals
- Large number of degrees of freedom acts like a "bath"

But…

- Readily access non-perturbative, "non-quasiparticle" regimes!

Higher dimensional extension





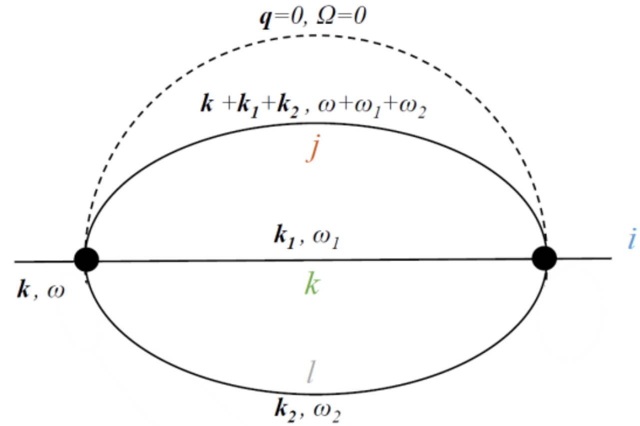
$$H = \sum_{\boldsymbol{k}} \sum_{i=1}^{N} \varepsilon_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}i} c_{\boldsymbol{k}i} + \frac{1}{N^{3/2}} \sum_{\boldsymbol{r}} \sum_{ijkl=1}^{N} J_{ijkl} c^{\dagger}_{\boldsymbol{r}i} c^{\dagger}_{\boldsymbol{r}j} c_{\boldsymbol{r}k} c_{\boldsymbol{r}l}$$

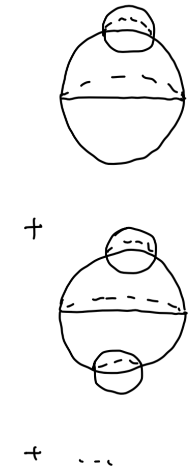
Same U_{ijkl} on every site $\bar{J}_{ijkl} = 0, \overline{J^2_{ijkl}} = J^2$

Other lattice generalizations of SYK: ***Parcollet, Georges; Gu, Qi, Stanford; Song, Jian, Balents; C. Xu et al; Patel, Sachdev et al.;…***

Electron self-energy

$$\Sigma(\mathbf{k},\omega) =$$





$\Sigma(\omega_n) \sim i\,\frac{J}{W}\omega_n$ ($\Sigma \ll W$; "correlated FL") $\qquad \omega_n, T \ll \frac{W^2}{J}$

$\Sigma(\omega_n) \sim i\sqrt{J|\omega_n|}\,\mathrm{sgn}(\omega_n)$ ($\Sigma \gg W$; "Incoherent Metal") $\qquad \omega_n, T \gg \frac{W^2}{J}$

D. Chowdhury, Y. Werman, EB, T. Senthil, PRX (2018)

Extended strange metallic behavior from SC puddles

Jörg Schmalian (KIT)

Noga Bashan

Evyatar Tulipman

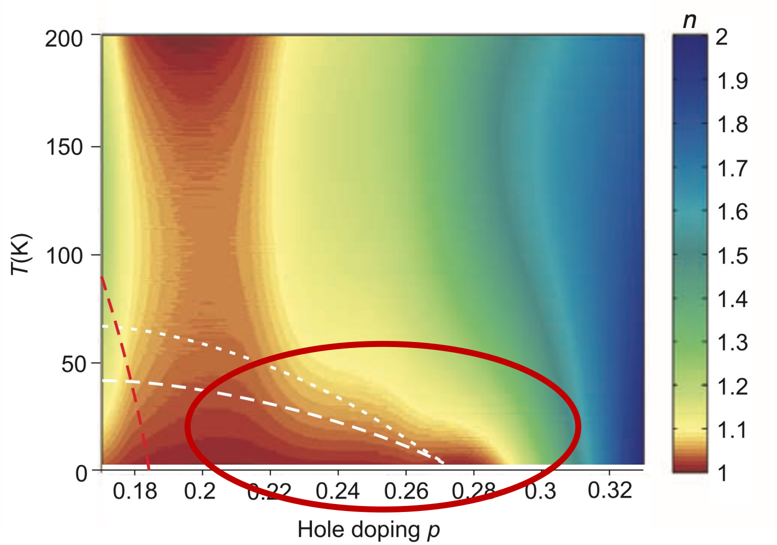
(WIS)

Steve Kivelson

Akshat Pandey

(Stanford)

Strange transport in correlated metals



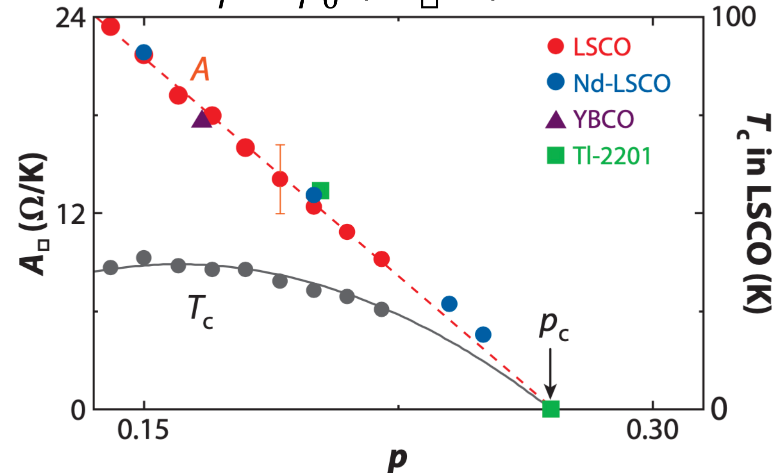
$La_{2-x}Sr_xCuO_4$

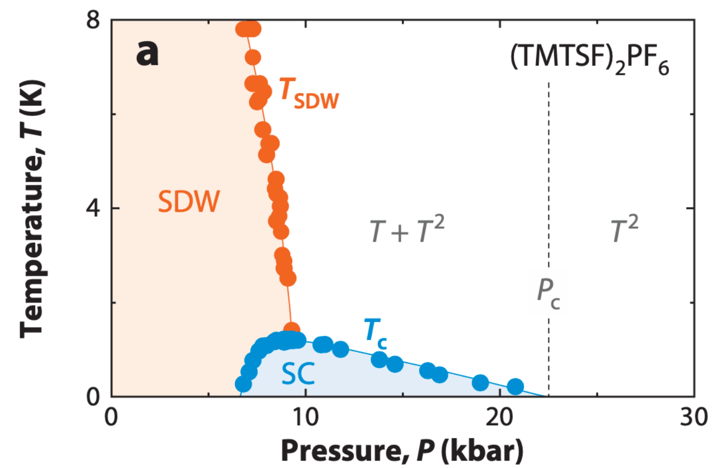
Cooper,…, Hussey (09')

$\rho \propto T$ over extended doping range: Critical *phase*?

Linear in T regime terminates approximately at the end of SC dome

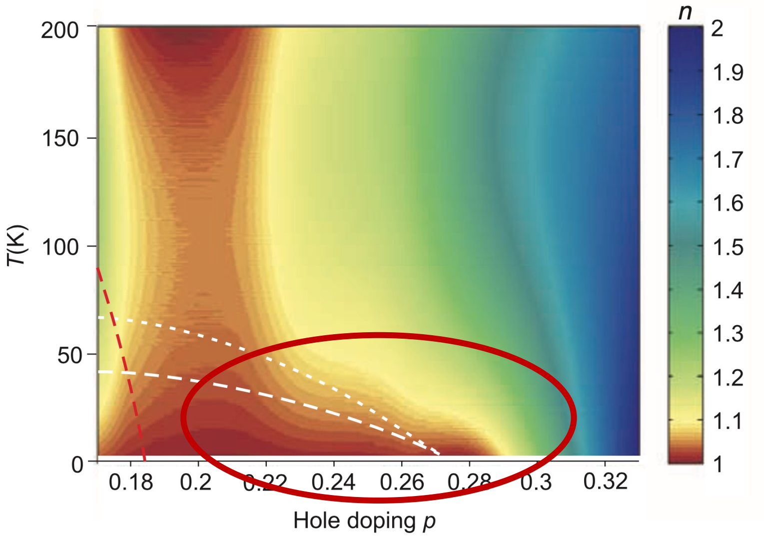
$$\rho = \rho_0 + A_\square T + BT^2$$





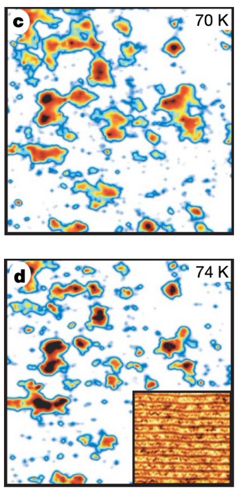
Review: Taillefer (10')

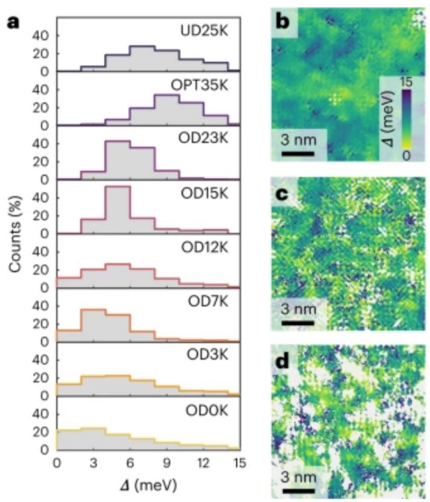
Superconducting “puddles” in overdoped cuprates



Superconducting "puddles" in overdoped cuprates

Gap inhomogeneity
revealed in STM

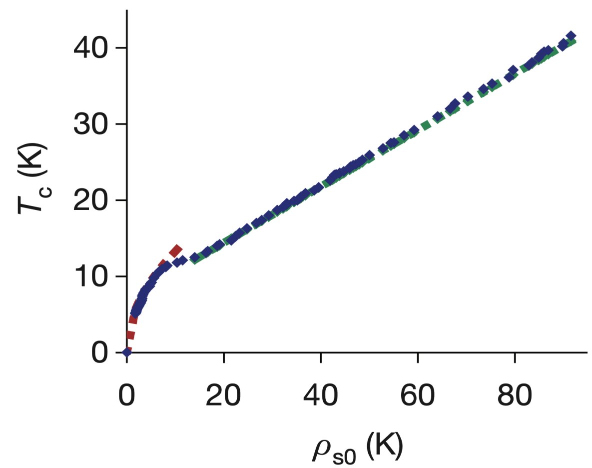




K. Gomez ..., A. Yazdani, Nature (07')

W. Tromp, ..., M. Allan, Nature Mat. (23')

Superfluid density
(overdoped LSCO)



I. Bozovic,..., A. T. Bollinger (16')

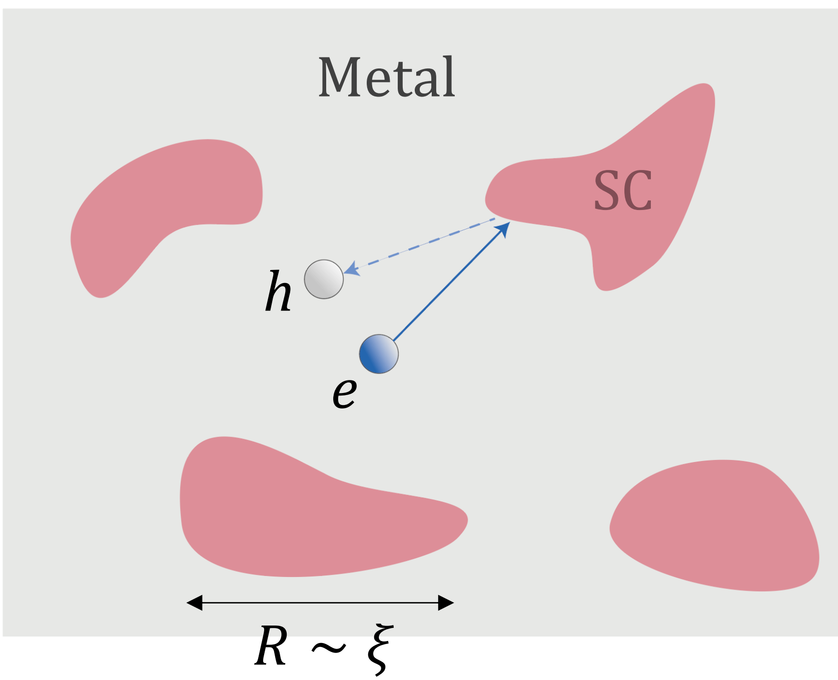
Further evidence: optical conductivity, specific heat

H.-H. Wen (04'); J. Wade (94'); F. Mahmood,..., P. Armitage (19')

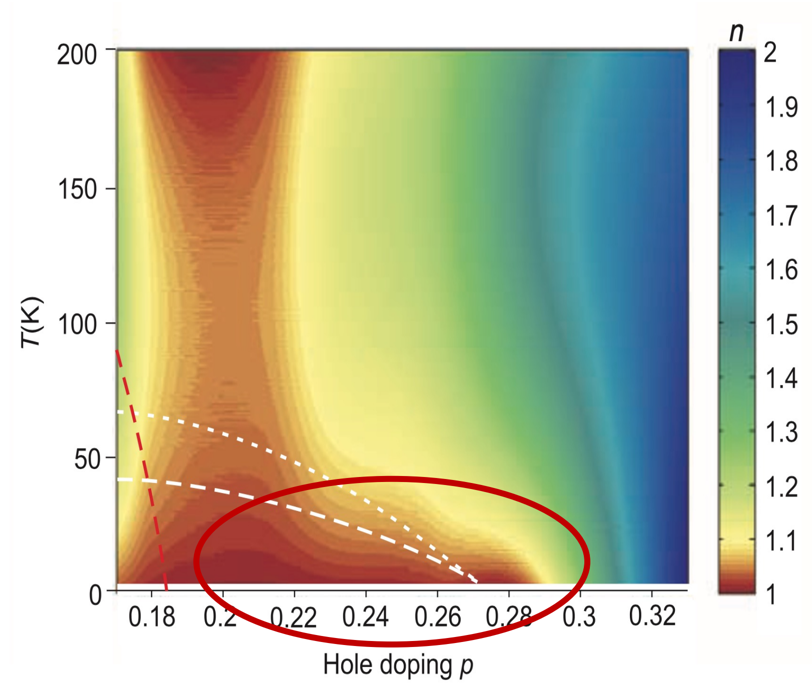
Theory: Z.-X. Li, S. Kivelson, D.-H. Lee (21')

Superconducting “puddles”

Inelastic Andreev scattering



Extended $\rho \propto T$ regime?



Cooper,…, Hussey (09’)

Model

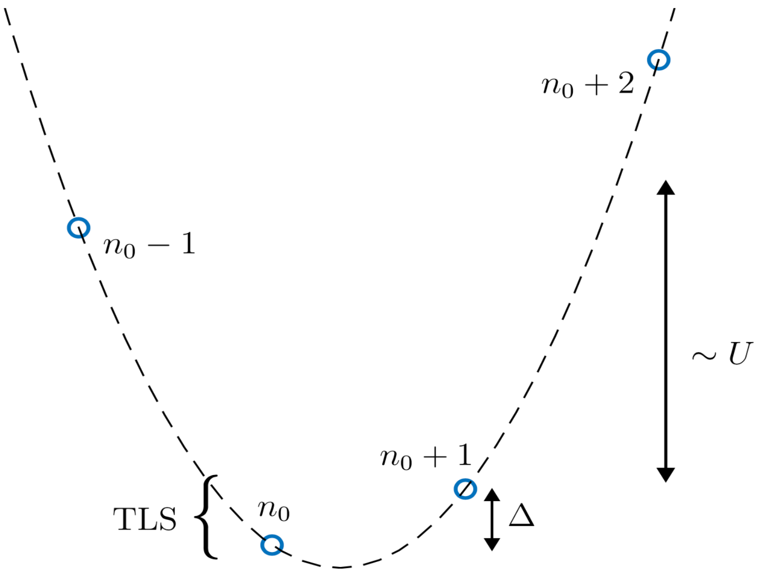
$$H = H_{\mathrm{el}} + H_{\mathrm{pud}} + H_{\mathrm{int}}$$

$$H_{\mathrm{el}} = \sum_{k,s} \epsilon_k c^{\dagger}_{k,s} c_{k,s}$$

$$H_{\mathrm{pud}} = \frac{U}{2}\left(\hat{n} - \frac{\Delta}{U}\right)^2, \; [\hat{\theta}, \hat{n}] = i$$

$$H_{\mathrm{int}} = \sum_{k,k'} g_{\perp}(k,k') e^{i\hat{\theta}} c_{k,\uparrow} c_{k',\downarrow} + h.c.$$

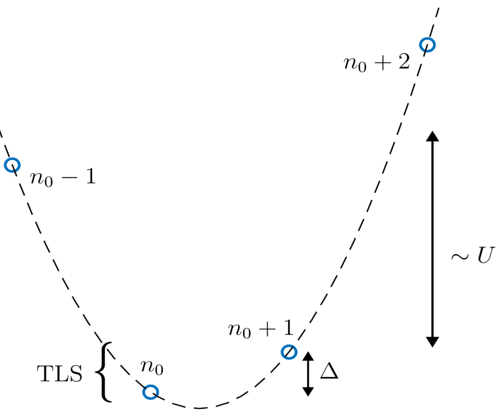
$$+ \sum_{k,k',s} g_z(k,k')\, \hat{n}\, c^{\dagger}_{k,s} c_{k',s}\,,$$



Model

At low energies, $E \ll U_r$ (*renormalized* charging energy):
Project to two low-lying levels

$$H_{\text{pud}} \to -\frac{\Delta}{2}\sigma^z,$$

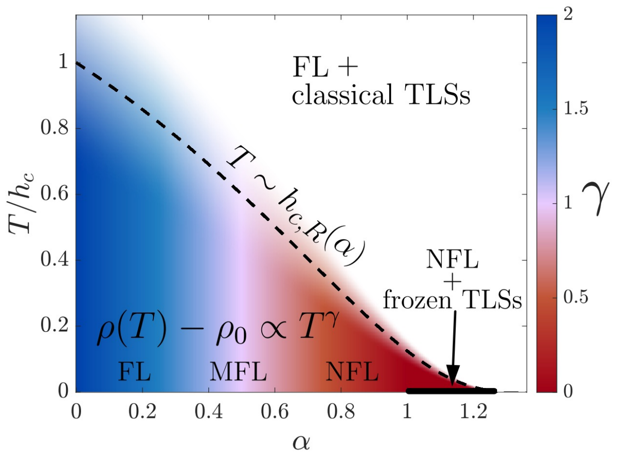


$$H_{\text{int}} \to \sum_{k,k'} g_\perp(k,k')\sigma^+ c_{k,\uparrow} c_{k',\downarrow} + h.c.$$

$$+ \sum_{k,k',s} \frac{g_z(k,k')}{2}\,\sigma^z\, c^\dagger_{k,s} c_{k',s}\,,$$

$$H_{\text{pud}} \to -\frac{\Delta}{2}\sigma^z,$$

C.f.: electrons interacting with localized (charge-neutral) two-level systems:



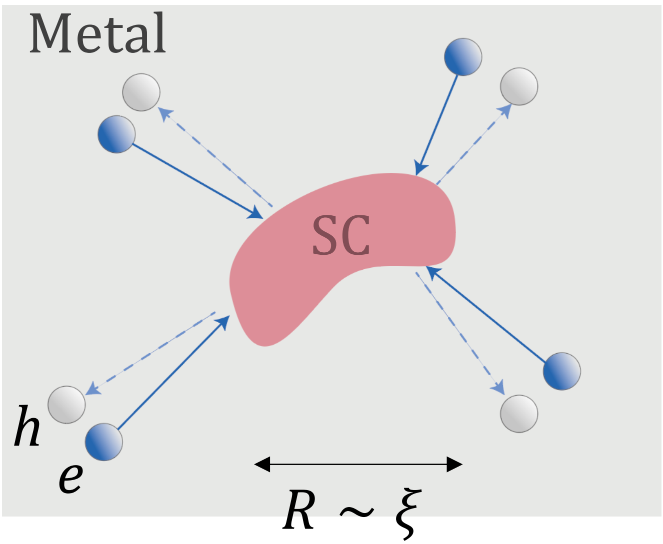
N. Bashan, E. Tulipman, J. Schmalian, EB, PRL (24')

Large-N limit and saddle point equations

$$H_{\text{int}} = \sum_{k,k'} g_{\perp}(k,k')\sigma^{+}c_{k,\uparrow}c_{k',\downarrow} + h.c.$$
$$+\sum_{k,k',s} \frac{g_z(k,k')}{2}\,\sigma^z\, c^{\dagger}_{k,s}c_{k',s}\,,$$

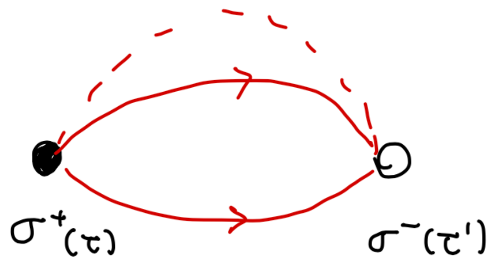
Treat the matrices $g_{\perp,z}(k,k')$ as random with a large dimension N

Physically, $N \sim k_F R$



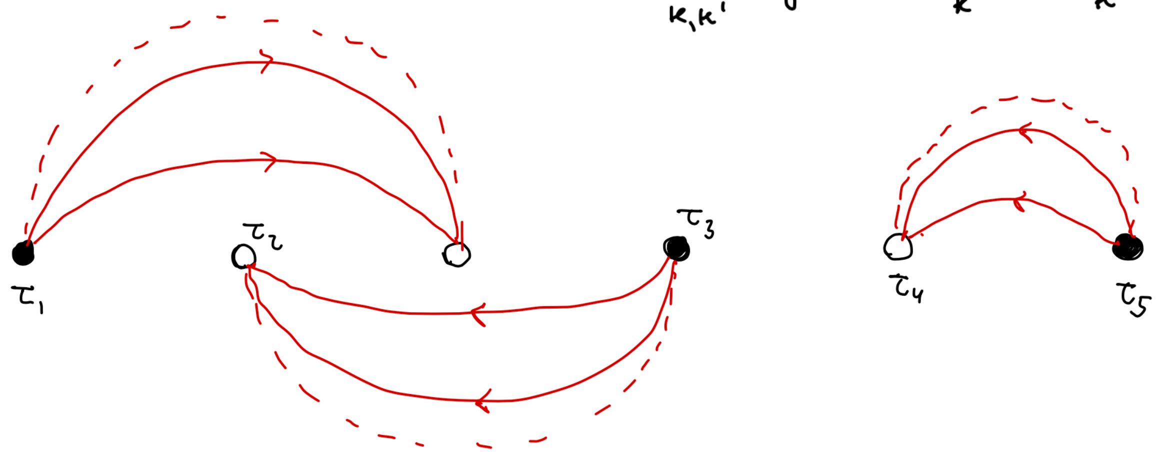
Large-N limit and saddle point equations

Expand the action in $g(k,k')$



$$\simeq \quad \sigma^+(\tau)\,\sigma^-(\tau)\;\Pi(\tau-\tau')$$

$$\Pi(\tau) = \sum_{k,k'} \overline{|g(k,k')|^2}\, G_k(\tau)\, G_{k'}(\tau) \sim \frac{1}{\tau^2}$$



Spin coupled to harmonic bosonic bath with correlator $\Pi(\tau)$

"Spin-boson" problem!

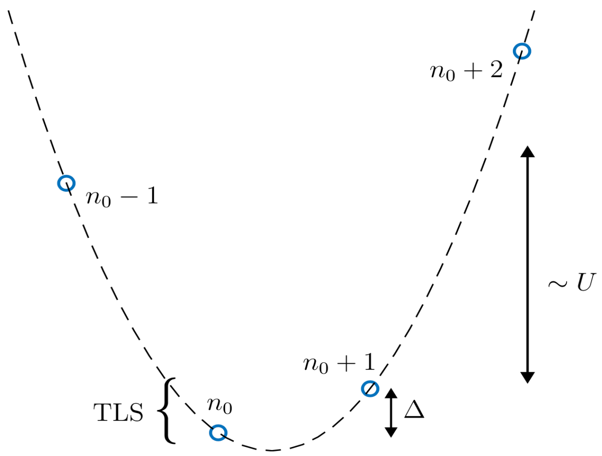
Model

$U_r \sim Ue^{-\alpha_\perp}$ Dimensionless coupling: $\alpha_\perp = \sum_{k,k'} \frac{|g_\perp|^2}{v_k v_{k'}}$

RG equations:*

$$\partial_\ell \alpha_\perp = -2\alpha_\perp^2$$
$$\partial_\ell \Delta = (1 - 2\alpha_\perp)\Delta$$

$\alpha_\perp$ marginally irrelevant



Δ uniformly distributed in $[-U_r, U_r]$

Marginal Fermi liquid:

$$\chi''(\Omega) = \overline{\langle \tau^+ \tau^- \rangle_\Delta} \sim \frac{n_\mathrm{p}}{U_r} \max\left(\frac{|\Omega|}{\mathrm{T}}, 1\right) \mathrm{sgn}(\Omega)$$

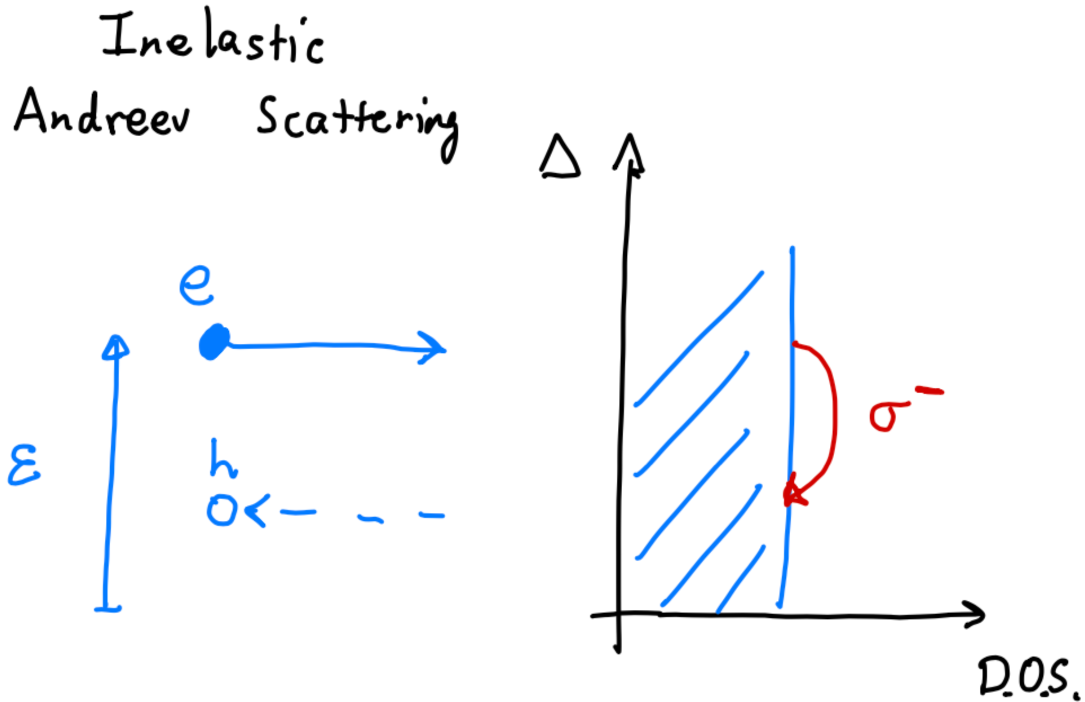
$$\rho(T) \sim \frac{h}{e^2} \frac{\alpha_\perp n_\mathrm{p}}{U_r} T$$

$$\mathrm{c}(T) \sim T \ln(U_r/T)$$

n_p: concentration of SC puddles

***Ignoring α_z**

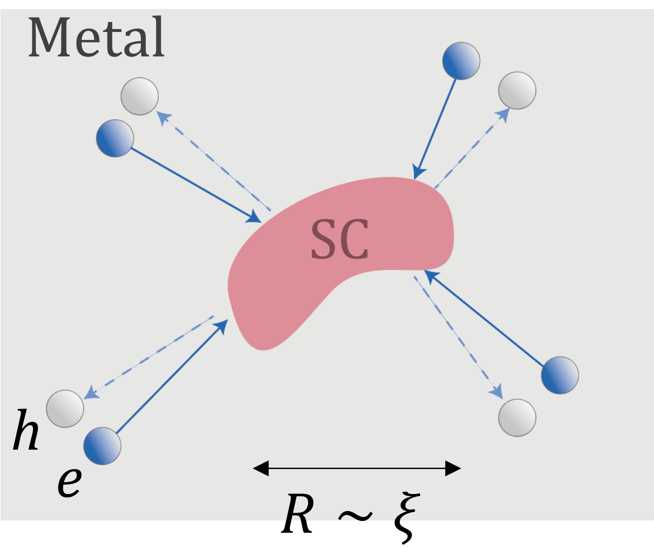
Why linear in *T*?

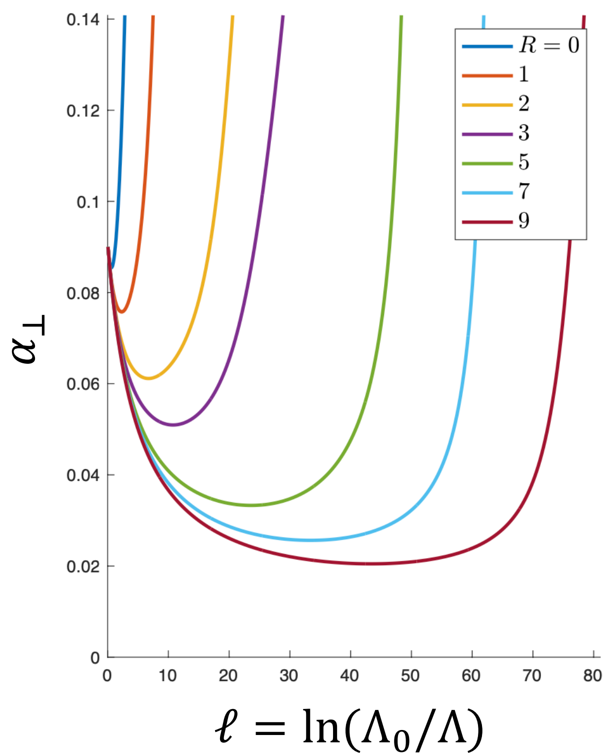


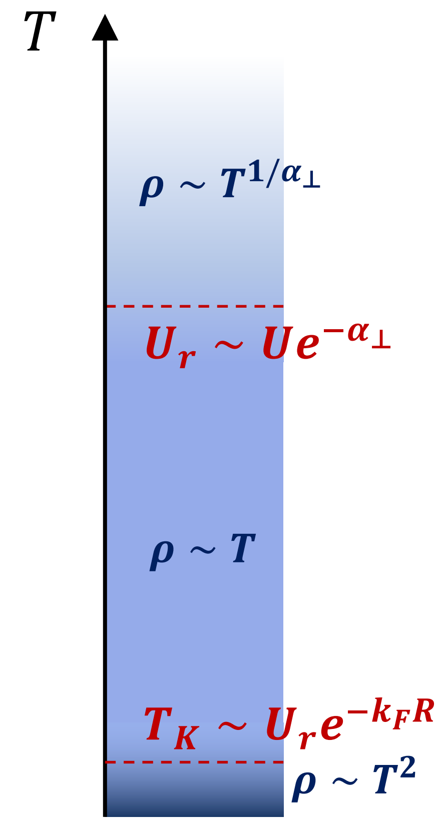
$$\frac{1}{\tau} \propto \max(\varepsilon, T)$$

Low-T breakdown (finite-N corrections)

At low T, the SC impurity is “Kondo screened”







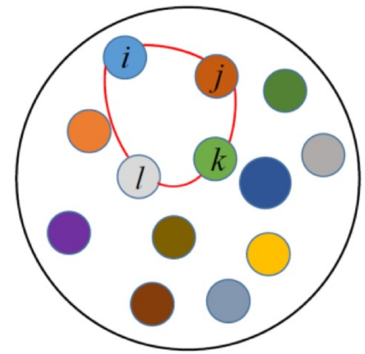
# of electronic channels:
$\sim k_F R$

Charge Kondo scale exponentially suppressed in droplet size

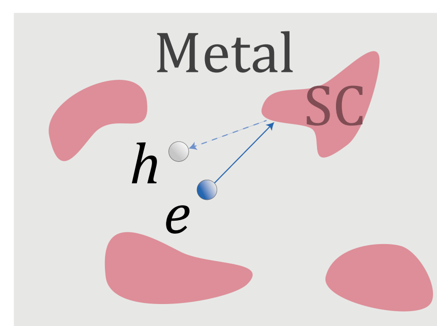
”Infinite-channel Kondo problem” (random $g_{k,k'}$):
G. Zarand, G. Zimanyi, F. Wilhelm (00’)

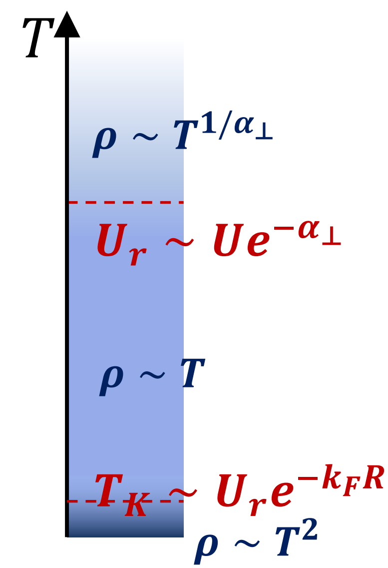
Summary

Route to solvability:
Make it random, make *N* large!



Strange metal from
coupling to SC puddles?

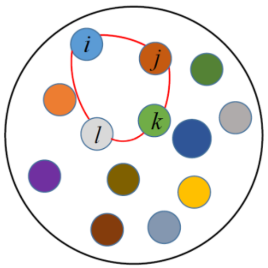


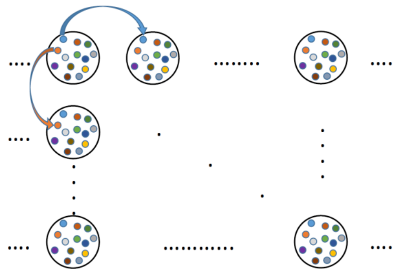


Thank you!

Model for a marginal Fermi liquid

Model: lattice of Sachdev-Ye-Kitaev (SYK) dots





$$H_c = \sum_{i=1,\boldsymbol{k}}^{N} \varepsilon_{\boldsymbol{k}} c_{\boldsymbol{k}i}^{\dagger} c_{\boldsymbol{k}i} + \sum_{ijkl=1,\boldsymbol{r}}^{N} \frac{U_{ijkl}}{N^{3/2}} c_{\boldsymbol{r}i}^{\dagger} c_{\boldsymbol{r}j}^{\dagger} c_{\boldsymbol{r}k} c_{\boldsymbol{r}l} + \sum_{ij=1,\boldsymbol{r}}^{N} \frac{W_{ij,\boldsymbol{r}}}{N^{1/2}} c_{\boldsymbol{r}i}^{\dagger} c_{\boldsymbol{r}j}$$

$U_{ijkl} = 0,\ \overline{U_{ijkl}^2} = U^2$ **Translationally invariant** in every realization

"Kondo lattice": Two bands c, f with bandwidths $W_f \ll W_c$

$$H = H_c + H_f + \sum_{ijkl=1,\boldsymbol{r}}^{N} \frac{V_{ijkl}}{N^{3/2}} c_{\boldsymbol{r}i}^{\dagger} c_{\boldsymbol{r}j} f_{\boldsymbol{r}k}^{\dagger} f_{\boldsymbol{r}l}$$

D. Chowdhury, Y. Werman, EB, T. Senthil, PRX (2018)

See also: A. Patel, J. McGreevy, D. Arovas, S. Sachdev, PRX (2018)